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Bulent Unel
Louisiana State University

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Department of Economics
Louisiana State University
Baton Rouge, LA 70803-6306
http://www.bus.lsu.edu/economics/

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Bulent Unel*
Louisiana State University

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Abstract

This paper develops a two-country, two-sector model of trade where the only difference between two countries is the cost of human capital formation. It is shown that this difference completely shapes the pattern of trade. Trade, in turn, affects the distribution of human capital both at extensive and intensive margins. Furthermore, cross-country differences in the costs of human capital formation determine the effects of trade on income distribution and welfare in each country. The paper also shows that lowering the cost of human capital acquisition in one country affects human capital formation, income distribution, and welfare asymmetrically in two countries.

JEL Classification: D7, F1, L1, and O1
Keywords: Human Capital, Income Inequality, Majority Voting, Trade

*Department of Economics; Louisiana State University; Baton Rouge, LA 70803, USA. Tel: (225)578-3792 and Fax: (225)578-3708. E-mail: bunel@lsu.edu. I am indebted to Shankha Chakraborty, Elias Dinopoulos, and Yoto Yotov for their valuable comments and suggestions.

1 Introduction

The standard Heckscher-Ohlin trade theory emphasizes the differences in factor endowments across countries as a key determinant for trade. Since this theory takes the aggregate factor endowments as exogenously fixed, trade has no impact on factor endowments. However, this contradicts the common belief that trade affects not only the sectoral composition, but also human capital formation across countries (O'Rourke and Williamson, 2005; Galor, 2011). In addition, some recent studies have shown that the distribution/dispersion of human capital across workers can also be an important source of comparative advantage even though countries may have similar aggregate endowments (Bougheas and Riezman, 2007; Bombardini et al., 2012).

This paper develops a two-country, two-sector competitive model of trade to study the interaction between human capital and trade. Labor is the only factor of production, and one sector (agriculture) uses only unskilled labor, whereas the other sector (manufacturing) uses workers with varying human capital (skill). Workers in agriculture earn the same wage rate, but the earnings of workers in manufacturing are proportional to their skill. The decision to become a skilled labor is endogenous, and the cost of human capital acquisition depends on individuals innate ability (characterized by a common distribution) as well as country-specific factors. The two countries are symmetric in every aspect except for the differences in the costs of human capital formation.

The model has several interesting predictions. First, it shows that cross-country differences in the costs of human capital acquisition shape the pattern of trade: under free trade the country with lower cost of human capital acquisition (Home) exports the manufacturing good, whereas the other country (Foreign) exports the agricultural good. However, unlike the traditional models, trade in turn has an effect on the distribution of human capital in each country: the fraction of individuals who choose to acquire human capital and their per capita level of human capital increase in Home, whereas they decrease in Foreign. Second, trade liberalization raises the income inequality between skilled and unskilled labor

in Home, while lowering it in Foreign. Free trade, however, always improves the aggregate welfare in both countries. Since not all agents gain from trade, the paper also investigates the conditions under which trade between the two countries become possible if the final decision in each country is based on majority voting. Finally, the model is also suitable to study the effects of a change in the cost of human capital formation in one country on both economies. The analysis shows that a reduction in one country's cost of human capital formation induces more individuals to acquire human capital, raises income inequality, and improves aggregate welfare in this country, while having opposite effects in the country's trade partner.

In an influential paper, Findlay and Kierzkowski (1983) incorporate human capital formation into the two-factor, two-good model of trade to study patterns of trade. In their model, individuals acquire human capital over time through education which uses physical capital and time. Although their model highlights the interaction between human capital formation and trade, the pattern of trade is ultimately driven by the cross-country differences in the aggregate capital-labor ratio as in the standard trade theory.[1] In this paper, aggregate endowments are the same across countries, and focuses on the distributional aspects of the cost of human capital formation.

Bond (1986) presents a *small* open economy model where firms are heterogeneous due to differences in managerial ability as in Lucas (1978) to study the extent to which the results of the standard trade theorems hold. Ishikawa (1996) incorporates workers' heterogeneity in the form of different human capital into the Heckscher-Ohlin model to examine patterns of trade and migration. Although individuals choose to become skilled or unskilled labor in these models, individuals don't choose to acquire human capital, and thus trade does not have any impact human capital formation. In addition, countries in these models differ in aggregate endowments.

[1] Borsook (1987) extends the Findlay and Kierzkowski model by considering individuals with different abilities. However, as in the Findlay and Kierzkowski model, differences in the aggregate factor endowments across countries determine the pattern of trade.

This paper is related to the recent literature that emphasizes the differences in distributions of factor endowments in explaining the pattern of comparative advantage. Grossman and Maggi (2000) present a model of trade between countries with similar aggregate factor endowments to study how the differences in dispersions of skill across the workers of the two countries determine the pattern of trade.[2] Bougheas and Riezman (2007) also show that the differences in human capital distributions can determine the pattern of trade between two otherwise symmetric countries.[3] However, unlike the present paper, the distributions of human capital are exogenous in these studies.

This paper is also related to the literature that studies the impact of trade on growth and productivity. Dinopoulos and Segerstrom (1999) develop a Schumpeterian growth model with trade in which individuals choose to become skilled worker, and they show that trade results in skill upgrading. In their model, countries are symmetric and markets are imperfectly competitive. Galor and Mountford (2008), using a unified growth theory model, also study the dynamic interaction between trade and human capital. They argue that trade has been the major factor behind the distribution of income, human capital, and population across the countries. This paper differs from theirs in several aspects; but most notably, the differences in production technologies across countries (as in the Ricardian model of trade) combined with parents' investment in their children's education play key factors in their model. In the present model, the production technology used in each sector is the same across countries, and individuals acquire human capital by themselves.

The rest of this paper is organized as follows. Section 2 introduces the model and studies the equilibrium in autarky. Section 3 characterizes the equilibrium in open economy, and investigates inequality and welfare effects of trade. It also studies the impact of a change in the cost of human capital formation in one country on inequality and welfare in each

[2]Building on the Grossman and Maggi model, Bombardini et al. (2012) empirically show that countries with a more heterogeneous workforce specialize in industries characterized by higher substitutability of skills.

[3]Yeaple (2005) studies how the interaction between technology adoption, trade, and workers' heterogeneity gives rise to firm heterogeneity. In his model, countries are symmetric, firms compete with each other in a monopolistic fashion, and increasing returns to scale is the driving factor for trade.

country. Section 4 concludes.

2 Setup of the Model

I begin by considering the equilibrium analysis under autarky to highlight the main points of the model. Consider a country that produces two goods, an agricultural good (a) and a manufacturing good (m) using labor as the only factor of production.[4] There is a continuum of individuals with constant mass of one ($L = 1$), each having different ability to acquire human capital. Ability levels (indexed by θ) are drawn from a common distribution $g(\theta)$ with cumulative distribution $G(\theta)$. The supply of skilled and unskilled labor is endogenously determined based on individuals ability levels and country-specific factors.

2.1 Demand

Individuals have identical preferences over the two goods as described by the following utility function

$$u = \left(\frac{c_a}{1-\alpha}\right)^{1-\alpha} \left(\frac{c_m}{\alpha}\right)^{\alpha}, \tag{1}$$

where c_i denotes consumption of good $i = a, m$; and $\alpha \in (0, 1)$ is an exogenous, constant parameter.

Denote by e the income of the individual, the utility maximization problem then yields

$$c_i = \alpha_i e / p_i, \tag{2}$$

where p_i is the price of good i, and $\alpha_a = 1 - \alpha$ and $\alpha_m = \alpha$. In the subsequent analysis, the agricultural good is chosen as numeraire so that its price equals one (i.e., $p_a = 1$), and for notational simplicity, the price of manufacturing good is denoted by p. Substituting the above consumption levels into (1) yields the following indirect utility function

$$\mathrm{v} = p^{-\alpha} e. \tag{3}$$

[4]Physical capital is assumed away for the sake of simplicity, and its incorporation into the model along the lines of Bond (1986) yields qualitatively similar results.

2.2 Production

The agricultural sector uses unskilled labor, and one unit of (unskilled) labor is required to produce one unit of agricultural good. Since this good is chosen as numeraire, the production technology ensures that the wage rate of unskilled labor equals one, i.e., w = 1.

Workers in the manufacturing sector are heterogenous with respect to their human capital. Production technology is constant returns to scale such that a worker with h units of human capital produces h units of good m.[5] Each worker is also the owner of his own production, and thus the revenue (gross income) generated by a worker with human capital h is ph. However, acquisition of human capital is costly. The cost of human capital acquisition of a worker with ability θ is

$$z(h, \theta) = \frac{b}{2}\frac{h^2}{\theta}, \tag{4}$$

measured in terms of the numeraire good. The parameter $b > 0$ represents a country-specific, exogenous constant.[6]

Since each individual's welfare increases with their income, workers wishing to acquire human capital maximize their net income, $e(\theta) = ph - z(h; \theta)$, by setting

$$h = \frac{\theta p}{b}. \tag{5}$$

This equation indicates that more able individuals acquire more human capital. Furthermore, the human capital level decreases in b and increases in p. Substituting h from equation (5) into $e(\theta) = ph - z(h; \theta)$ yields

$$e(\theta) = \frac{\theta p^2}{2b}. \tag{6}$$

Thus, the income of an individual working in manufacturing sector increases in his ability θ and the price p, decreases in the cost parameter b.

[5] That is, $y_m(h) = h$. Assuming that $y_m(h) = h^{\beta} l^{1-\beta}$ so that the manufacturing sector uses both skilled and unskilled workers does not change results.

[6] Considering a more general cost function $z = [f + bh^2/2]p^{\beta}$ (where f is a fixed parameter and $\beta \in (0, 1)$ is a constant) so that the investment in human capital uses both goods, makes exposition more complicated without changing the main results.

Given that labor is mobile between the two sectors, an individual chooses to work in manufacturing if and only if $e(\theta) \geqslant \mathrm{w} = 1$. The critical ability level, θ^*, at which an individual is indifferent for working between the two sectors is determined by $e(\theta^*) = 1$. Hence,

$$\theta^* = \frac{2b}{p^2}. \tag{7}$$

Any individual with $\theta \geqslant \theta^*$ invests in human capital and works in manufacturing sector, while individuals with $\theta < \theta^*$ work in agriculture and earns $e(\theta) = 1$. According to (7), the ability cutoff θ^* increases in the cost parameter b; decreases in the relative price of manufacturing p.

The total output produced by each sector then is

$$Y_a = G(\theta^*), \tag{8a}$$

$$Y_m = \int_{\theta^*}^{\infty} h(\theta)g(\theta)d\theta = \mathbb{E}[\theta \geqslant \theta^*]p/b, \tag{8b}$$

where $\mathbb{E}[\theta \geqslant \theta^*] = \int_{\theta^*}^{\infty} \theta g(\theta)d\theta$ (and recall that the supply of labor is normalized to one). Furthermore, substituting h from (5) into the cost function (4) yields $z(\theta) = \theta p^2/2b = \theta/\theta^*$, where the last equality follows from (7). Aggregating over the corresponding individuals yields the total amount of agricultural good used in the human capital formation:

$$Z_a = \frac{\mathbb{E}[\theta \geqslant \theta^*]}{\theta^*}. \tag{9}$$

Given the cutoff θ^*, the income distribution and welfare across individuals can easily be characterized. Substituting p from (7) into the income per capita equation (6) implies that $e(\theta) = \theta/\theta^*$ for all $\theta \geqslant \theta^*$. Thus, the per capita income (or expenditure) is

$$e(\theta) = \begin{cases} 1 & \text{if } \theta < \theta^* \\ \theta/\theta^* & \text{if } \theta \geqslant \theta^* \end{cases} \tag{10}$$

The aggregate expenditure, E, then is

$$E = \int e(\theta)g(\theta)d\theta = G(\theta^*) + \frac{\mathbb{E}[\theta \geqslant \theta^*]}{\theta^*}. \tag{11}$$

Finally, using (10) in the indirect utility function (3) yields the distribution of welfare across individuals

$$\mathrm{v}(\theta, p) = \begin{cases} p^{-\alpha} & \text{if } \theta < \theta^* \\ p^{-\alpha}\theta/\theta^* & \text{if } \theta \geqslant \theta^* \end{cases} \tag{12}$$

Since each individual's welfare depends on his own income, the way aggregate (social) welfare is measured becomes important in investigating the impact of trade on welfare. The aggregate welfare is measured as the sum of individuals' utilities:[7]

$$\mathbb{V} = \int \mathrm{v}(\theta, p) g(\theta) d\theta = p^{-\alpha} E, \tag{13}$$

where the last equality directly follows from (10) and (11).

2.3 Closed Economy Equilibrium

In autarky, $C_a = Y_a - Z_a$ and $C_m = Y_m$, where C_i is the aggregate consumption of good $i = a, m$. Equation (2) then implies that $pY_m/(Y_a - Z_a) = \alpha_m/(1 - \alpha_m)$, where α_m is given by (1). Substituting Z_as from (9) into the latter and using (7) yields

$$G(\theta^*) = \frac{2 - \alpha}{\alpha} \frac{\mathbb{E}[\theta \geqslant \theta^*]}{\theta^*}. \tag{14}$$

The left–hand side (LHS) is increasing in θ^*, whereas the right–hand side (RHS) is decreasing in it. Thus, there exists a unique cutoff θ^* that solves equation (14). In addition, the ability cutoff level θ^* is independent from the cost parameter b.

Lemma 1. *There exists a unique ability cutoff level θ^* that satisfies equation (14).*

Consider now two countries (Home and Foreign) that are identical in terms of preferences, production technologies, and the distributions of ability. However, the cost parameter b_j ($j = H, F$) differs across countries. Note that the equilibrium condition (14) indicates

[7] The welfare analysis based on the welfare function (12) is permissible if the economy admits a representative agent. Note that $\mathrm{v}(p, e) = ep^{-\alpha}$ implies that the indirect utility function of each individual is in the Gorman form; as a result, individuals' preferences can be aggregated and represented by those of a representative agent. See Chapter 4 in Mas-Colell et al. (1995) for more on the aggregation of preferences.

that both countries have the same ability cutoff in autarky, i.e. $\theta_H^{*a} = \theta_F^{*a} \equiv \theta^{*a}$ (with superscript a denoting autarky). Assuming that $b_H < b_F$, it then follows from equation (7) that the relative price of manufacturing good in autarky is cheaper in Home, i.e., $p_H^a < p_F^a$. This difference in the relative price of the manufacturing good across countries creates an opportunity to trade.

3 The Open Economy

Consider a world of two countries ($j = H, F$) which are symmetric in every aspect except for the cost parameter b_j. Without loss of generality, assume that $b_H < b_F$ so that (ceteris paribus) the cost of human capital acquisition is cheaper in Home. The cross-country differences in the costs of human capital formation can arise from the differences in the quality of institutions, the access to credit and training, labor market frictions, and so on. This section investigates the impact of trade between these two countries on human capital formation, income distribution, and welfare in each country.

Trade between the two countries is free and this ensures that both countries face the same price for each good. In addition, workers in the agricultural sector earn the same wage across both countries, i.e., $\mathrm{w}_H = \mathrm{w}_F = 1$. The model is closed with two additional assumptions. First, trade is balanced:

$$(Y_{aj} - Z_{aj} - C_{aj}) + p(Y_{mj} - C_{mj}) = 0, \qquad j = H,\, F, \tag{15}$$

where Z_{aj} is given by (9). Second, the global demand for each good ($i = a,\, m$) must be equal to its (net) global supply:

$$C_{iH} + C_{iF} = Y_{iH} + Y_{iF} - \mathbb{I}_i(Z_{aH} + Z_{aF}), \tag{16}$$

where $\mathbb{I}_i$ is an indicator variable that equals one if $i = a$, and zero otherwise.

3.1 Open Economy Equilibrium

Note that the balanced trade condition (15) implies that $E_j = C_{aj} + pC_{mj} = Y_{aj} - Z_{aj} + pY_{mj}$. Using equations (8a), (8b), and (9) then implies that E_j is still given by equation (11) with θ^* is replaced by θ_j^*.

To determine the cutoff level θ_j^*, the market clearing condition (16) will be used. Substituting $C_{aj} = \alpha_a E_j$ and $Y_{aj} - Z_{aj}$ into equation (16) for $i = a$, and rearranging the terms yields

$$\sum_j G(\theta_j^*) = \frac{2-\alpha}{\alpha} \sum_j \frac{\mathbb{E}[\theta \geqslant \theta_j^*]}{\theta_j^*}. \tag{17}$$

Since θ_j^* is a decreasing function of p as indicated by equation (7), the LHS is decreasing in p, whereas the RHS is increasing in it. Thus, the above equation yields a unique solution for p. Substituting this back into equation (7) then implies a unique solution for θ_j^*. In addition, since it is assumed that $b_H < b_F$, it then follows that $\theta_H^* < \theta_F^*$. The following lemma summarizes these results.

Lemma 2. *Assume that $b_H < b_F$. There exist unique ability cutoffs θ_H^* and θ_F^* that satisfy the equilibrium condition (17). In addition, under free trade Home's ability cutoff is smaller than Foreign's, i.e. $\theta_H^* < \theta_F^*$.*

How do these cutoff levels compare to those in autarky? As discussed in the previous section, the ability cutoffs in autarky are the same in both countries. Since $\theta_H^* < \theta_F^*$ under free trade, it then follows that trade liberalization decreases the ability cutoff in Home, while increasing it in Foreign. This further suggests that in equilibrium Home should export manufacturing good, and Foreign agricultural good. This is indeed the case, and to see this consider the net supply of agricultural good $Y_{aj} - Z_{aj} - C_{aj}$. Using equations (2), (8a), (9) and (11) implies

$$Y_{aj} - Z_{aj} - C_{aj} = \alpha G(\theta_j^*) - (2-\alpha)\frac{\mathbb{E}[\theta \geqslant \theta_j^*]}{\theta_j^*}. \tag{18}$$

Using $\theta_H^* < \theta_F^*$ in (18) then implies that $Y_{aH} - Z_{aH} - C_{aH} < Y_F - Z_{aF} - C_{aF}$. This combined with the market clearing condition (16) implies that $Y_{aH} - Z_{aH} - C_{aH} < 0$ and $Y_{aH} - Z_{aH} - C_{aH} > 0$. That is, Home exports the manufacturing good and imports the agricultural good. Thus, the pattern of trade is determined by the differences in the costs of human capital acquisition.

The result that $\theta_H^* < \theta_H^{*a}$ ($\theta_F^* > \theta_F^{*a}$) indicates that trade liberalization increases (decreases) the fraction of individuals who acquire human capital in Home (Foreign). Furthermore, since the relative price of the manufacturing good increases after trade liberalization, equation (5) implies that the amount of human capital acquired by each individual in Home is greater than that in autarky. The opposite happens in Foreign. Thus, trade liberalization increases (decreases) human capital in Home (Foreign) both at extensive and intensive margins.

Proposition 1. *Consider two countries (Home and Foreign), each producing agricultural and manufacturing goods. The only difference between the two countries is their costs of human capital acquisition, and assume that (ceteris paribus) the cost is cheaper in Home (i.e., $b_H < b_F$). Moving from autarky to free trade*

a. lowers the ability cutoff level in Home, raises it in Foreign (i.e., $\theta_H^* \downarrow$, $\theta_F^* \uparrow$);

b. makes Home export manufacturing good, and Foreign export agricultural good;

c. increases human capital in Home both at extensive and intensive margins, while having an opposite effect on it in Foreign.

The intuition behind these results is simple. Exposure to trade increases the relative price of the manufacturing good in Home, while decreasing it in Foreign. This makes manufacturing more (less) attractive in Home (Foreign), and thereby inducing more (less) individuals to work in this sector. Since individuals have identical preferences across countries, the expansions in Home's manufacturing and Foreign's agriculture makes Home export

manufacturing good and Foreign export agricultural good. In addition, increased income in Home's manufacturing induces workers in this sector to acquire more human capital.

The next step is to investigate the impact of trade on income inequality. The income distribution under free trade is still characterized by equation (11) with θ_j^*. A comparison of this distribution with that under autarky reveals that individuals with $\theta > \theta^{*a}$ have experienced an increase in income by $\theta/\theta_H^* - \theta/\theta^{*a}$, individuals with $\theta \in (\theta_H^*, \theta^{*a}]$ have experienced an increase in income by $\theta/\theta_H^* - 1$, whereas the income of all other individuals has remained the same. In addition, note that $e(\theta)/e(\theta') = \theta/\theta' > 1/\theta'$, with $\theta \in (\theta_H^*, \theta_H^{*a}]$ and $\theta' > \theta_H^{*a}$. Thus, trade lowers the income inequality between individuals with $\theta \in (\theta_H^*, \theta^{*a}]$ and those with $\theta > \theta^{*a}$ in Home. Since $\theta_F^* > \theta^{*a}$, free trade has an opposite impact on the income distribution in Foreign. The following proposition summarizes these results.

Proposition 2. *Consider two countries, Home and Foreign, as described in this section. Trade liberalization increases the income inequality between skilled and unskilled workers in Home, while reducing it in Foreign. In addition, trade lowers the income inequality between individuals with $\theta \in (\theta_H^*, \theta^{*a}]$ and those with $\theta > \theta^{*a}$ in Home, whereas it raises the income inequality between individuals with $\theta \in (\theta_F^*, \theta^{*a}]$ and those with $\theta > \theta^{*a}$ in Foreign.*

3.2 Welfare

Using equation (12) implies that the welfare change, $\triangle \mathrm{v}_j = \mathrm{v}_j(\theta, p) - \mathrm{v}_j(\theta, p_j^a)$, is

$$\triangle \mathrm{v}_H = (p_H^a)^{-\alpha} \times \begin{cases} \left(\frac{\theta_H^*}{\theta^{*a}}\right)^{\alpha/2} - 1 & \text{if } \theta \leqslant \theta_H^* \\ \frac{\theta}{\theta_H^*}\left(\frac{\theta_H^*}{\theta^{*a}}\right)^{\alpha/2} - 1 & \text{if } \theta \in [\theta_H^*, \theta^{*a}] \\ \frac{\theta}{\theta_H^*}\left(\frac{\theta_H^*}{\theta^{*a}}\right)^{\alpha/2} - \frac{\theta}{\theta^{*a}} & \text{if } \theta \geqslant \theta^{*a} \end{cases}$$

$$\triangle \mathrm{v}_F = (p_F^a)^{-\alpha} \times \begin{cases} \left(\frac{\theta_F^*}{\theta^{*a}}\right)^{\alpha/2} - 1 & \text{if } \theta \leqslant \theta^{*a} \\ \left(\frac{\theta_F^*}{\theta^{*a}}\right)^{\alpha/2} - \frac{\theta}{\theta^{*a}} & \text{if } \theta \in [\theta^{*a}, \theta_F^*] \\ \frac{\theta}{\theta_F^*}\left(\frac{\theta_F^*}{\theta^{*a}}\right)^{\alpha/2} - \frac{\theta}{\theta^{*a}} & \text{if } \theta \geqslant \theta_F^* \end{cases}$$

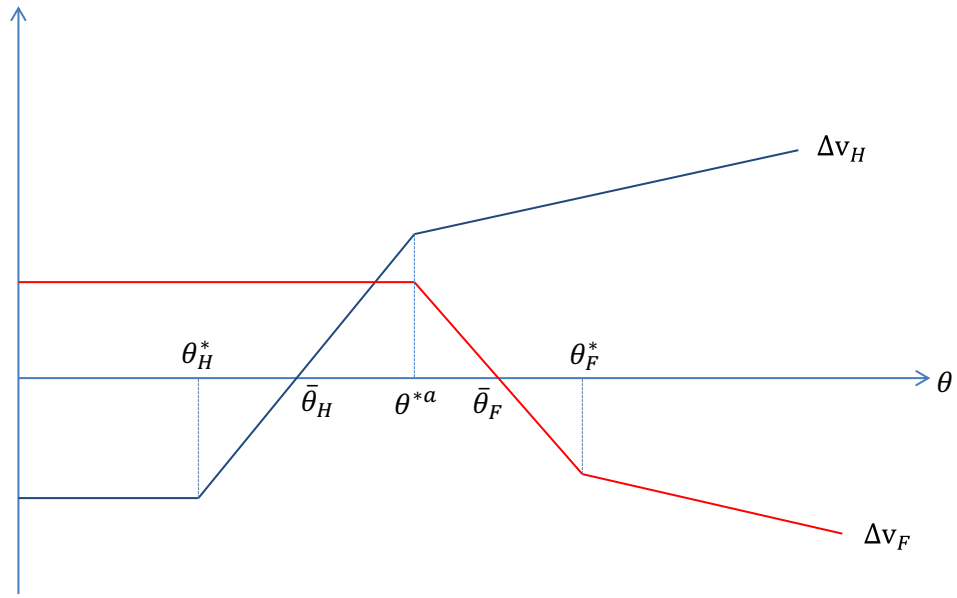

Figure 1: Change in welfare for each individual in each country.

In obtaining the above expressions, $p/p_j^a = (\theta^{*a}/\theta_j^*)^{1/2}$, which follows from equation (7), is used. The above equations are plotted in Figure 1. Accordingly, trade liberalization reduces the welfare of all Home's workers with $\theta \in [1, \bar{\theta}_H)$, whereas it increases the welfare of all workers with $\theta > \bar{\theta}_H$. In Foreign, the welfare of all workers with $\theta \in [1, \bar{\theta}_F)$ increases, but the welfare of workers with $\theta > \bar{\theta}_F$ falls. Since $\triangle \mathrm{v}_j(\bar{\theta}_j) = 0$, it then follows that

$$\bar{\theta}_j = \theta_j^{*\frac{\alpha}{2}} (\theta^{*a})^{1-\frac{\alpha}{2}}. \tag{19}$$

To determine the impact of trade on welfare, consider the aggregate welfare function given by (13). The welfare function (13) is convex, and the necessary condition for minimizing $\mathbb{V}_j$ yields the equilibrium condition (14).[8] Thus, trade is welfare improving. The following proposition summarizes these results.

Proposition 3. *Consider two countries (Home and Foreign), each producing agricultural and manufacturing goods. The only difference between the two countries is their costs of*

[8]Equation (7) implies that $dp/d\theta_j^* = -p/2\theta_j^*$. Using this in $d\mathbb{V}_j(p, \theta_j^*)/d\theta_j^* = 0$ yields $\alpha G(\theta_j^*) = (2 - \alpha) \int_{\theta_j^*}^{\infty} (\theta/\theta^*) g(\theta) d\theta$, which is the same as the equilibrium condition (14).

human capital acquisition, and assume that (ceteris paribus) the cost is cheaper in Home (i.e., $b_H < b_F$). Moving from autarky to free trade

a. *raises (lowers) the welfare of all individuals with $1 \leqslant \theta < \bar{\theta}_H$ ($1 \leqslant \theta < \bar{\theta}_F$), but it lowers (raises) the welfare of all individuals with $\theta > \bar{\theta}_H$ ($\theta > \bar{\theta}_F$) in Home (Foreign).*

b. *improves the aggregate welfare in both countries.*

Although trade increases aggregate welfare, not all agents gain from the free trade. Under what conditions does trade become a viable option? To answer this question, I make two further assumptions. First, I assume that a country follows a free trade policy if the median voter prefers free trade to autarky. In other words, a country chooses to trade if the majority of its population approves it. This approach is reminiscent of Mayer's (1984) seminal work.

Second, I assume that ability levels follow a Pareto distribution defined over $[1, \infty)$

$$G(\theta) = 1 - \theta^{-k}, \tag{20}$$

where $k > 1$ is the shape parameter. Using Pareto distribution makes analysis more tractable and has been used in similar context (e.g., Buera et al., 2011; Helpman et al., 2011). In addition, equation (7) ensures that the corresponding human capital (and thus, firms' productivity) levels also follow a Pareto distribution. This is consistent with many empirical studies that have shown that firm sizes follow Pareto distribution (see, e.g., Helpman et al., 2004).

Using the above distribution function, the aggregate welfare function (13) then becomes,

$$\mathbb{V}_j = (2b_j)^{-\frac{\alpha}{2}} \left[\theta_j^{*\frac{\alpha}{2}} + \frac{\theta_j^{*\frac{\alpha}{2}-k}}{k-1} \right], \tag{21}$$

and attains its (global) minimum at[9]

$$\theta^{*a} = \left[\frac{2k-\alpha}{\alpha(k-1)}\right]^{1/k}. \tag{22}$$

Using equations (7) and (17) together with the distribution function (20), it is straightforward to show that under free trade the ability cutoffs are given by

$$\theta_H^* = \left[\frac{1+\mathbf{b}^k}{2\mathbf{b}^k}\right]^{\frac{1}{k}} \theta^{*a}, \qquad \theta_F^* = \mathbf{b}\theta_H^*, \qquad \mathbf{b} \equiv \frac{b_F}{b_H}, \tag{23}$$

where θ^{*a} is given by (22). Note that θ_H^* decreases in $\mathbf{b}$, whereas θ_F^* increases in it.

Since all Home workers with $\theta > \bar{\theta}_H$ benefit from free trade (see Figure 1), Home chooses to trade under majority voting if and only if $G(\bar{\theta}_H) < 0.5$. Equations (19) and (23) then yield

$$G(\bar{\theta}_H) < 0.5 \quad \Leftrightarrow \quad \mathbf{b} > \gamma \equiv \left\{2\left[\frac{2\alpha(k-1)}{2k-\alpha}\right]^{\frac{2}{\alpha}} - 1\right\}^{-1/k}. \tag{24}$$

Since $2\alpha(k-1) < 2k-\alpha$, it then follows that $\gamma < 1$,[10] which further implies that $b > \gamma$ always holds. Thus, Home, based on majority voting, always chooses to trade.

Foreign, on the other hand, chooses to trade under majority voting if and only if $G(\bar{\theta}_F) > 0.5$. Following the same steps as in the previous case yields

$$G(\bar{\theta}_F) > 0.5 \quad \Leftrightarrow \quad \mathbf{b} > 1/\gamma,$$

where $\gamma < 1$ is defined in (24). Thus, trade between these countries, based on majority voting, is possible if and only if $b > 1/\gamma$.

Proposition 4. *Consider two countries (Home and Foreign), each producing agricultural and manufacturing goods. Suppose that $b_F > b_H$ so that (ceteris paribus) the cost of human capital formation is cheaper in Home. The two countries, based on majority voting, choose to trade with each other if b_F is sufficiently larger than b_H.*

[9] This directly follows from Proposition 3. Alternatively, differentiating (21) with respect to θ_j^* also yields (22). Note that second-order condition is satisfied, since $d^2\mathbb{V}/d\theta_j^{*2} \mid_{\theta^{*a}} = k\alpha(2b_j)^{-\alpha/2}(\theta^{*a})^{\alpha/2-2}/2 > 0$. Furthermore, one can show that $d\mathbb{V}/d\theta_j^* < 0$ for $\theta_j^* < \theta^{*a}$ and $d\mathbb{V}/d\theta_j^* > 0$ for $\theta_j^* > \theta^{*a}$.

[10] It is further assumed that the parameters α and k are chosen high enough so that $\gamma > 0$.

3.3 Change in the Cost of Human Capital Acquisition

The analysis so far has focused on how trade liberalization affects allocation of skills across sectors and its effects on income distribution and welfare in each country. Another interesting question to address is the implications of a change in the cost parameter b in one country on the income distribution and welfare in each country when countries freely trade with each other. The following lemma characterizes the impact of this change on the ability cutoff level in each country (see Appendix for the proof).

Lemma 3. *A reduction in b_j decreases the ability cutoff θ_j^* in country j, while increasing it in the other country.*

Without loss of generality, consider a reduction in Foreign's cost parameter b_F. According to the above lemma, such a reduction decreases the ability cutoff level θ_F^* in Foreign, while increasing θ_H^* in Home. Thus, the fraction of individuals who acquire human capital increases (decreases) in Foreign (Home). Furthermore, equations (5) and (7) indicate that each worker in manufacturing in Foreign will acquire more human capital. The opposite will be observed in Home. Using equation (10) then implies that the income inequality between manufacturing and agricultural workers increases in Foreign, while it decreases in Home. As shown in Appendix B, such a reduction in b_F will improve Foreign's welfare, while reducing Home's. The following proposition summarizes the results.

Proposition 5. *Consider two freely trading countries (Home and Foreign), each producing agricultural and manufacturing goods. A reduction in the cost of human capital acquisition (in the form of lowering b) in one country increases human capital both at extensive and intensive margins, raises the income inequality between skilled and unskilled workers, and improves the welfare in this country, while having completely opposite effects on these variables in the other country.*

A reduction in b_F makes human capital formation more affordable in Foreign, which in turn induces more people to work in manufacturing. That is, the ability cutoff necessary to work in manufacturing falls in Foreign. This will increase individual level human capital formation as well as the net income per capita for workers in Foreign's manufacturing. An expansion in Foreign's manufacturing sector lowers the relative price of manufacturing good as well. This in turn makes human capital formation more costly for some workers in Home, and thus they prefer to work in agricultural sector. In this case, the ability cutoff θ_H^* will raise in Home.

A reduction in the ability cutoff in Foreign increases each manufacturing worker's income (see equation (10)), which in turn increases the aggregate income in Foreign. This, combined with a reduction in the price of manufacturing, improves the aggregate welfare in Foreign. On the other hand, a fall in the price of manufacturing combined with the lower human capital reduces each manufacturing worker's income in Home. This in turn reduces the aggregate income in Home. Although reduction in the price of the manufacturing good has a positive effect on welfare, reduction in the aggregate income overcomes this effect, and hence, the aggregate welfare decreases in Home.

4 Concluding Remarks

This paper has studied the interaction between human capital formation and trade using a two-sector, two-country model of trade in which individuals choose to become either an unskilled worker and work in agriculture or skilled worker and work in manufacturing. The only difference between the two countries is that the cost of human capital acquisition is cheaper in Home. The analysis shows that this difference determines the pattern of comparative advantage across countries. However, trade also affects the distribution of human capital in each country.

The preferences are in the Cobb-Douglas form in the present model. As is known from Engel's law, income elasticity of demand for food (agricultural good) is less than one. To

capture this, I extend my model by considering Stone-Geary preferences in which there is a subsistence consumption requirement for agricultural good. Analysis based on this extension yields a smaller manufacturing sector in each country, i.e. the ability cutoff in each country is higher than that in the benchmark model. However, other results remain unchanged and are available upon request.

The present model assumes that trade is free. One possible extension is to introduce a tariff and investigate the income inequality and welfare implications of a government policy that uses the revenue from tariffs to make human capital formation more affordable for workers. Using this extended framework to analyze how the educational policy determined by different legislative systems (e.g., social planner vs. majority voting) may uncover interesting results. The model also assumes a well-functioning financial markets for individuals who wish to acquire human capital. Incorporating credit market imperfections in financing the cost of human capital formation is another extension.

Appendix

A. Proof of Lemma 3

Without loss of a generality, suppose that there is a reduction in b_F. Totally differentiating (17) with respect to b_F yields

$$\sum_j A_j \frac{d\theta_j^*}{db_F} = 0, \qquad A_j \equiv g(\theta_j^*) + \frac{2-\alpha}{2}\frac{\mathbb{E}[\theta \geqslant \theta_j^*]}{\theta_j^{*2}}. \tag{25}$$

Differentiating (7) with respect to b_F, on the other hand, yields

$$\frac{d\theta_H^*}{db_F} = -\frac{2\theta_H^*}{p}\frac{dp}{db_F}, \qquad \frac{d\theta_F^*}{db_F} = \frac{2}{p^2} - \frac{2\theta_F^*}{p}\frac{dp}{db_F}. \tag{26}$$

Substituting these into (25) and rearranging terms yields

$$\frac{dp}{db_F} = \frac{A_F}{p\sum_j \theta_j^* A_j} > 0. \tag{27}$$

It then follows from (26) that $d\theta_H^*/db_F < 0$. Substituting (27) into the the second equation in (26), on the other hand, yields

$$\frac{d\theta_F^*}{db_F} = \frac{2\theta_H^* A_H}{p^2(\theta_H^* A_H + \theta_F^* A_F)} > 0. \tag{28}$$

B. Proof of Proposition 5

Without loss of generality, consider again a reduction in b_F. Since this reduction lowers p, equation (5) immediately implies that h will decrease in Home. Totally differentiating equation (5) with respect to b_F yields

$$\frac{dh_F}{db_F} = \frac{\theta}{b_F}\frac{dp}{db_F} - \frac{\theta p}{b_F^2}.$$

Substituting (27) into the above equation and using (7) yields

$$\frac{dh_F}{db_F} = \frac{\theta}{b_F p}\left[\frac{1}{\theta_H^* A_H/A_F^* + \theta_F^*} - \frac{2}{\theta_F^*}\right] < 0,$$

where A_j is given by (25).

To determine the welfare impact, I use the aggregate welfare function $\mathbb{V}_j = p^{-\alpha}E_j$, where E_j is given by (11). Differentiating (11) with respect to b_F and using (26) yields

$$\frac{d\mathbb{V}_j}{db_F} = -p^{-1-\alpha}\left[\alpha G(\theta_j^*) - (2-\alpha)\frac{\mathbb{E}[\theta \geqslant \theta_j^*]}{\theta_j^*}\right]\frac{dp}{db_F},$$

where the term in the square bracket represents the net demand for manufacturing good in country j as indicated in equation (18). This term is negative for Home, and positive for Foreign as shown in Proposition (2). Since $dp/db_F > 0$, it follows that $d\mathbb{V}_H/db_F > 0$ and $d\mathbb{V}_F/db_F < 0$.

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