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Abstract

Several current issues in economics are centered on scheduling and matching problems, notably including the 2012 Nobel Prize winning work. Such problems usually lie outside the scope of most undergraduate courses. The authors present a relatively simple problem that can be used to introduce the graph theory needed to teach these interesting but somewhat difficult topics.

Keywords: Search, Matching, Networks, Graph Theory, Operations Research

JEL Codes: C44, D47, D83, D85

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1 Introduction

While economist Bill Harbaugh was on sabbatical in Aix-en-Provence, one of his daughters attended a French school. It was convention that each day before class all students would kiss each other. Harbaugh took this bedrock of French culture and turned it into a simple problem. He posed the following question: How fast can a group of 35 students can kiss each other, given that they only have 10 minutes between classes? (Harbaugh 2003) With this question, Harbaugh opens the door to find and conceptualize an efficient way to solve the 'kissing problem'. In this paper we first briefly mention an earlier attempt to solve this question. We then provide a more efficient solution and link it to a historically well known problem. Finally, we illustrate how this problem and its framework can be used to introduce topics like matching, networks, and search that are typically not covered in undergraduate classes despite their growing importance. This highlights the pedagogical importance of Harbaugh's quandary about efficient kissing.

The notions used here can be used to discuss numerous concepts in class like relating to graph theory, game theory, networks, scheduling, matching and search. The majority of these topics are usually covered in advanced or specialized undergraduate classes in Economics with some topics being covered in Operations Management. Although we discuss specific instances later, we believe that the problem discussed here and its many variants can be used to introduce difficult concepts in classes like Intermediate Macro, Money and Banking as well as in courses like Advanced Macro, Game Theory, Development Economics, Networks, Mathematics for Economists, Experimental Economics at the undergraduate level. It can also be used to introduce abstracts concepts in MBA and in Operations Research courses.

In the paper *Having Fun with Organized Kissing: A Pedagogical Note* (Wei, 2008), Jong-Shin Wei provides an answer to Harbaugh's question. Let us assume that the students kiss each other only once. Since we have 35 students kissing each other before class, this results in a total of $C_2^{35} = 595$ kisses. Let us quickly recapitulate the approach Wei suggests in his paper. He claims that the 35 French students can kiss each other before class in $2n-3$ rounds, where n is the number of students. He defines the *maximum load* to be $\frac{n}{2}$ if n is even and $\frac{n-1}{2}$ if n is odd where *load* is the number of kisses taking place at each point in time. Assuming as Harbaugh suggests that we need to assign 5 seconds per kiss the organized kissing exercise will take 5.6 minutes. According to Wei, this can be accomplished by having all the students sit in a circle. Then starting in a clockwise fashion student number 1 gets up, walks to student 2, kisses her and then walks to student 3, and

so on. When student 1 moves to student 4, student 2 should get up and move to student 3, and both 1 and 2 should continue moving. This continues with each student getting up when they and the next student clockwise have been kissed by all the prior students. This leaves an upper bound of $2n-3$ as it takes $n-1$ for student 1 to reach student n, which will be n's first kiss. Student n will then need $n-2$ rounds to kiss all the other students, giving us a total of $2n-3$ rounds.

We will first show that such organized kissing can be performed in less than $2n-3$ rounds. We show that it in fact only takes n rounds to complete the kissing ritual when n is odd, with a constant maximum load of $\frac{n-1}{2}$. When n is even, as it will become clear, all rounds will have a load of $\frac{n}{2}$, and it will take $n-1$ rounds to complete the kissing ritual. We then discuss the antecedents of this problem followed by its pedagogical applications.

2 Optimal Organized Kissing

In this section we begin by solving the original Harbaugh problem followed by the solution of the general case.

2.1 Solution to n=35

Instead of solving the problem in $2n-3$ or 67 rounds as proposed by Wei, the method we use accomplishes the same task in exactly $n = 35$ rounds. The difference stems from the way we organize the students – it guarantees maximum load per round cutting down the total number of rounds. To explain out method, we describe in detail how the first and last three rounds work. First, we divide the students into two rows. Since we have an odd number of students, there will always be one student who will not have a kissing partner. We assign this student the *pivot position*, in which she will not kiss anyone for the one round she occupies this position (see Figure 1 below). Denote student i by S_i.

In the very the first round, S_1 is assigned the pivot position, and in the second round, S_{35} is assigned this position. Once we have the students aligned into two rows, with the top row students facing the bottom row students (except for the pivot student who is not facing anyone), the students will proceed and kiss their assigned counterpart. So, for the first round, S_2 kisses S_{35}, S_3 kisses S_{34}, and so on. As is evident from the figures, we always have a load of 17 students per round. This is the maximum possible load making the suggested solution the most efficient.[1]

[1] In the next section we discuss the origins of this result in detail. It follows from the celebrated Vizing's Theorem (Vizing, 1964) in graph theory.

Once the given round is over, students rotate clockwise. So for the second round, S_{35} will play the pivot position and S_1 will now participate in the kissing routine. We continue this process until all students have kissed each other, which ends in round 35. It is easy to check that after this round the students will in the same positions they occupied in the very first round.

Figure 1: How the optimal solution works

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	*Round 1*
	35	34	33	32	31	30	29	28	27	26	25	24	23	22	21	20	19	

35	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	*Round 2*
	34	33	32	31	30	29	28	27	26	25	24	23	22	21	20	19	18	

34	35	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	*Round 3*
	33	32	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	

$\vdots$

4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	*Round 33*
	3	2	1	35	34	33	32	31	30	29	28	27	26	25	24	23	22	

3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	*Round 34*
	2	1	35	34	33	32	31	30	29	28	27	26	25	24	23	22	21	

2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	*Round 35*
	1	35	34	33	32	31	30	29	28	27	26	25	24	23	22	21	20	

2.2 General Case

To solve the general case we need to consider two separate cases for determining the efficient schedule, depending on the number of participants. The simplest case is when n is odd. Then we form a wheel as described above, which will take n rounds to rotate fully. At each step we have a load of $\frac{(n-1)}{2}$, which is the maximal load, making this is the most efficient solution for an odd case. Each member kisses a different person in each step, excluding the step that they are in the pivot position, as the person who kisses no one creates an offset that allows the kissing of students who start both an odd and an even number of spaces away.

The solution when n is even is very similar. In this case, we form a wheel as in the odd case, made of $n-1$ participants. The final, n^{th} participant, will be held stationary, and will kiss the

person at the end of the wheel (who in the odd case has no one to kiss). Thus we have a load of $\frac{n}{2}$ in each round, which is maximal, and the person on the end kisses all other participants, while the other $n-1$ participants all kiss each other as above. Thus, for n even, we can complete the problem in $n-1$ rounds, as we will simply rotate the wheel as described above. The first two and last two rounds of this process are illustrated in Figure 2, with the stationary n^{th} individual highlighted in bold type.

These procedures are the most efficient possible, as they all involve a maximal load at every step, and in each case the rotations lead to non-repeating kisses.

Figure 2: How the optimal solution works in the even case

36	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	*Round 1*
		35	34	33	32	31	30	29	28	27	26	25	24	23	22	21	20	19	

36	35	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	*Round 2*
		34	33	32	31	30	29	28	27	26	25	24	23	22	21	20	19	18	

⋮

36	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	*Round 34*
		2	1	35	34	33	32	31	30	29	28	27	26	25	24	23	22	21	

36	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	*Round 35*
		1	35	34	33	32	31	30	29	28	27	26	25	24	23	22	21	20	

3 A Brief History of Organized Kissing

The earliest formulation of the problem Harbaugh (2003) puzzled over can be traced back to graph theory. Vadim G. Vizing, a Russian mathematician who is best known for his contributions to modern graph theory was the first to conceptualize and solve this problem. His result, subsequently known as Vizing's Theorem, was published in Russian in a Siberian journal in 1964. Vizing had largely been sidelined by the Soviet Union because of his German ancestry, with his family being forced to Siberia from Ukraine when he was 10. After completing his PhD, he attempted to return to Kiev; however was bared from residing there, and was limited to provincial towns in Ukraine. In fact, similar political pressure resulted in the rejection of his habilitation thesis and effectively banned him from attending conferences.

The idea for the theorem was born when Vizing started to work on a practical problem that involved the coloring of the wires of a network (Gutin and Toft, 2000). To solve the problem, he looked at a theorem by Claude Shannon[2] in which it is stated that the edges of any graph[3], including those with multiple edges between a given pair of nodes, can be colored using $3d/2$ colors, where d is the maximum numbers of edges any node has.[4] After studying Shannon's theorem, Vizing became interested in the subject and started looking at cases where graphs did not have multiple edges. He ended up improving Shannon's bound of $3d/2$ for this case stepwise.

After sending his results to the renowned Russian mathematics journal *Doklady* and being rejected, Vizing published his results in 1964 in a little known journal named *Metody Diskret. Analiz.* Interestingly, *Doklady* rejected his findings on the grounds that it was just an extension of Shannon's Theorem, when in reality it was a substantial improvement on it. Vizing himself did not think his work in graph theory was going to have the impact it currently has since when his work was published in 1964. Interestingly, he had some conjectures on edge coloring problems which remain unsolved to this day.

4 Having Fun With Organized Kissing: *Wei's Pedagogical Applications*

Although his solution to the problem was not efficient, Wei (2008) nevertheless proposes some pedagogical applications of this problem. He proposes using it in courses such as Operational Research, Quantitative Methods for Business, Mathematics for Economists and similar courses in which students could learn from a problem-solving demonstration. He suggests that given the nature of the solution, this approach can be used to understand how a barter economy works. When teaching courses like Money and Banking and introductory or intermediate Macroeconomics it can be used to demonstrate the need for different exchange rates. One other application he recommends is to illustrate how government intervention can be beneficial when it aims at eliminating a negative externality. Trying to organize a large group of people can create high transaction costs if a organizational method does not exist. In this case, government can help in organizing and highly reducing transaction costs. While the method suggested above will reduce transaction costs and

[2]This is the Shannon of the famous Shannon entropy as measure in Information Theory. See for instance (Wu 2003).

[3]Consider a set of friends that interact with each other on Facebook. In graph theory these individuals form the nodes and their Facebook relationships are represented by links or edges between them. Thus, a graph is a collection of nodes and edges, with the edges connecting the nodes.

[4]These edge coloring problems are related to the well known Four Color Theorem, which we elaborate on later.

be more efficient, in the following section we suggest some additional applications that follow quite naturally from the graph-theoretic nature of the problem.

4.1 Having More Fun with Organized Kissing: *More Pedagogical Applications*

This toy problem about the kissing ritual[5] can be used to introduce several other concepts. Networks is an emerging research area in economics.[6] It is a field that has come to the forefront in recent work, having been assigned its own JEL classification (D85) only in the last few years. Networks is an interdisciplinary field that borrows from mathematics, especially graph theory and game theory. Recall, that a graph is defined as a collection of nodes and edges or links between them. So in our problem each student is a node and each kiss a link or edge between two nodes. The behavior of the nodes which describes the relationships between them typically involve strategic aspects. In the simplest setting networks are purveyors of information. To explain information flow using the notion of networks, the students can be allowed to shake hands or hug in different patterns and examine how information transmission takes place. They can clearly see that the *complete network*[7], a network where everyone talks to everyone, is important in a situation when all individuals have different information. They then can be allowed to interact with each other without imposing a pattern or restriction and come up with a stable interaction pattern by themselves or pushed to one. Much like Wei's government intervention idea, the notion that interactions can be costly in terms of time or effort can also be introduced by letting them interact in different patterns. Using this framework, notions of stability like *Nash networks* (Bala and Goyal, 2000) or *pairwise stability* (Jackson and Wolinsky, 1996) can be introduced to students. For example when it is costly to talk to each other, they will quickly realize that in order to gather all the information from each other, the best types of networks are trees.[8] This is an example of a Nash network in the model of Bala and Goyal. Also the idea of socially optimal networks where society's welfare is maximized by the network can be introduced in this setting. Other example about trading favors, help or the role of networked relationships as an alternative to the market can be introduced using this basic setup.

[5] We suggest replacing kissing – a bedrock of French culture with an appropriate Americanism – either the polite but firm handshake or a hug!

[6] For an excellent introduction to this topic and the undergraduate level see the freely available book by Easley and Kleinberg (2010) which can be found at http://www.cs.cornell.edu/kleinber/

[7] Illustrations of a complete network, along with the other types of networks defined in this paper are provided at the end.

[8] A tree is a network which is no longer connected after the deletion of any links. In other words it is a network where every agent is connected to all the other agents using the fewest possible number of links. In the economics literature trees are also called minimally connected networks.

These types of examples can be introduced even in a Development Economics class which covers topics like mutual help and insurance. Students may form their own networks which will allow them to exchange favors or hugs, but will restrict the set of possible hugs only to people with whom they have links.

The kissing problem can be used to introduce more advanced concepts in graph theory, such as the edge coloring problem, that will be relevant for courses like Operations Research, Operations Management and Networks. An edge coloring graph G is an assignment of colors to the edges of G, one color to each edge, such that adjacent edges are assigned different colors. In our example, we are attempting to color a *complete graph*[9], where the vertices are the students, and the colors of the edges correspond to the rounds in which the connected pair of students kiss. The minimum number of colors that can be used is referred to as the edge chromatic number, $\chi_1(G)$. Vizing (1964) showed that if we know the maximum degree of a graph, then we are very close to knowing the edge chromatic number of the graph.

Vizing's Theorem. *The edges of a graph with maximum degree d can be colored in at most $d + 1$ colors so that no two edges with a common vertex are colored the same. Moreover, the edges of a p-graph with maximum degree d (where any two vertices are joined by at most p edges) can be colored in at most $d + p$ colors so that no two edges with a common vertex are colored the same.*

For a formal proof of this, see *An Introduction to Graph Theory* by Gary Chartrand and Zhang Ping (2005). Our optimal solution of the organized kissing problem follows from this. In a complete network, every individual or node is connected to all the others. Thus in a complete graph with n nodes, each node has degree $n - 1$. Then from Vizing's theorem it follows that either $n - 1$ or n colors will be required, corresponding to our cases with an even and an odd number of students respectively.

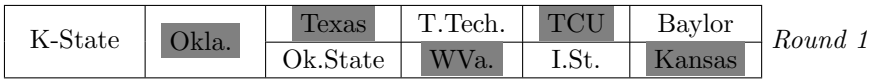
These concepts can then be used to introduce the celebrated *Four Color* problem. In this problem, an arbitrary map on a plane is given, and the regions must be colored such that no two adjacent regions have the same color. It is known that any such map requires at most 4 colors to allow such a coloring to be made.

However it can also be used to introduce topics that are very near and dear to students – the issue of scheduling. This may concern scheduling classes, exams or sporting events. Scheduling a round-robin tournament, for example, is equivalent to solving the edge-coloring problem on a

[9] A complete graph is one which contains a single edge between every pair of vertices.

complete graph, where each node is a team, and each color a date for the contest. As each node is only allowed one edge of any given color, this means that a team is not subject to conflicting matches on a given date, and the completeness of the graph means that each team plays every other team once. Thus scheduling a round-robin tournament is equivalent to Harbaugh's organized kissing problem, with the teams taking the place of the students, and the order of games being the rounds of kissing. These edge coloring problems are well studied in mathematics, where the focus is generally on finding efficient colorings, which means the fewest number of required colors (or the smallest $\chi_1(G)$) . This is equivalent to our efforts to create efficient schedules in the sense of requiring the shortest amount of time to complete a play.

As an illustration we apply our previous algorithm to the problems of scheduling round robin tournaments. To do so, we first establish a method of determining home and away teams for each match. Our method will be to shade the spaces of the home team at each step. In the odd case, we will create a checkerboard pattern, such that each team alternates home and away matches. In the even case, we will have a checkerboard pattern for the wheel portion of the algorithm, with the stationary participant alternating being the home or away team. This will cause each team to play either $\frac{n}{2}$ or $\frac{n}{2} - 1$ home games in the even case, which is the best possible as each team will play an odd number of games. For our example, we will use the Big12 conference, which currently has 10 teams playing a round robin schedule. Let us start with the following arrangement of teams:



By applying the same rotation as in the kissing problem, we obtain the following schedule:

Figure 3: A Round Robin Schedule

Week					
1	K-State@Okla.	Ok.St.@Texas	T.Tech@WVa.	I.St.@TCU	Baylor@Kansas
2	Ok.St.@K-State	WVa@Okla.	Texas@I.St.	Kansas@T.Tech.	TCU@Baylor
3	K-State@WVa.	I.St.@Ok.St.	Okla@Kansas	Baylor@Texas	T.Tech.@TCU
4	I.St.@K-State	Kansas@WVa.	Ok.St.@Baylor	TCU@Okla.	Texas@T.Tech.
5	K-State@Kansas	Baylor@I.St.	WVa.@TCU	T.Tech.@Ok.St.	Okla.@Texas
6	Baylor@K-State	TCU@Kansas	I.St@T.Tech.	Texas@WVa.	Ok.St.@Okla.
7	K-State@TCU	T.Tech.@Baylor	Kansas@Texas	Okla@I.St.	WVa.@Ok.St.
8	T.Tech.@K-State	Texas@TCU	Baylor@Okla.	Ok.St.@Kansas	I.St.@WVa.
9	K-State@Texas	Okla@T.Tech.	TCU@Ok.St.	WVa.@Baylor	Kansas@I.St.

One notable example of this is in classroom scheduling. If we take some nodes to be classrooms, and other nodes to be instructors, we can take the connections to be class times. Obviously classrooms are not connected to other classrooms, nor are instructors connected to other instructors. Thus this forms a *bipartite graph*, which is defined as a graph where the set of nodes can be split into two groups such that all edges connect one group to the other group. At any given time, a classroom can be occupied by at most one class, so each class time attached to a room must be distinct. An instructor can only be in one place at one time, so all class times attached to a given instructor must be unique. Thus, by solving this problem, we can find the minimal number of class times required for scheduling. In this case there is no clear reason why each instructor should be matched with each classroom, nor that an instructor shouldn't be assigned to the same classroom for multiple class periods, so we do not have a simple algorithm to generate the colorings as in the kissing ritual (this is in fact a difficult problem). This can thus show the potential for more challenging scheduling problems, which is especially valuable if this kissing problem is being used to introduce Operations Research to a class.

The framework of the kissing problem can be used to introduce the notion of search costs. Assume that in each time period the students can interact with one other person only. This interaction could occur in a random manner or in way that we describe in Figures 1 and 2. The time taken for every ideal match can then be used to explain the notion of search costs. Similarly, the students can also be exposed to notions of random matching versus pre-specified matching in advanced undergraduate course to introduce them to the notion of matching.

Most of the courses taught at the undergraduate level deal with markets where the allocation is done by prices. However, in many markets there are no prices to facilitate resource allocation. For example in the marriage market or the market for matching students and schools there are no prices to guide the outcomes. The Deferred Acceptance algorithms, which won Roth and Shapely the 2012 Nobel Prize, are fundamentally attempting to deal with this sort of search. In a sense this takes off from where Wei's suggestion about barter stops – it may not be possible to find a double coincidence of wants situation easily. In the absence of prices, time spent searching is costly, however there are large benefits from obtaining a better match. Attempted matches occur essentially randomly, until a good enough set of matches are found. The algorithm presented here organizes the initial attempts at matching, functioning as the "speed dating" version of such a market. By quickly running through initial encounters, everyone had a more accurate picture of options, and thus efficient matches can occur more rapidly. In particular, no deference would be

required if both sides know they have their best possible match from all available participants. For example if we separate into two groups, one running 1 through 18, and the other from 19 to 35, and want only matches across groups, we can accomplish this matching in 18 rounds, as shown in Figure 4.

Figure 4: How the optimal solution works for split groups

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	*Round 1*
	35	34	33	32	31	30	29	28	27	26	25	24	23	22	21	20	19	
2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	1	*Round 2*
	35	34	33	32	31	30	29	28	27	26	25	24	23	22	21	20	19	
3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	1	2	*Round 3*
	35	34	33	32	31	30	29	28	27	26	25	24	23	22	21	20	19	

⋮

17	18	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	*Round 17*
	35	34	33	32	31	30	29	28	27	26	25	24	23	22	21	20	19	
18	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	*Round 18*
	35	34	33	32	31	30	29	28	27	26	25	24	23	22	21	20	19	

This "kissing problem" provides a nice example of scheduling for an undergraduate game theory course. It would also be helpful for a course in experimental economics, where it provides some insight into how to organize the experiments. These scheduling problems are also a major focus of Operations Research, where this would be a nice introduction to some basic ideas in the field. Finally we refer the interested reader to a paper that explores disruptions in round-robin tournaments (Borkotokey, Sarangi, and Wiser (2012)). In this paper, the possibility of adding or removing contestants from a round-robin tournament that is already underway is discussed. Adding contestants requires the addition of enough extra rounds such that the remaining number of rounds the tournament will now take is equal to the number of rounds simply starting with number of contestants we now have would have taken, while removing contestants sometimes allows for a rescheduling of contests in later rounds to be found which reduces the total number of rounds required.

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5 Illustrations of Graphs

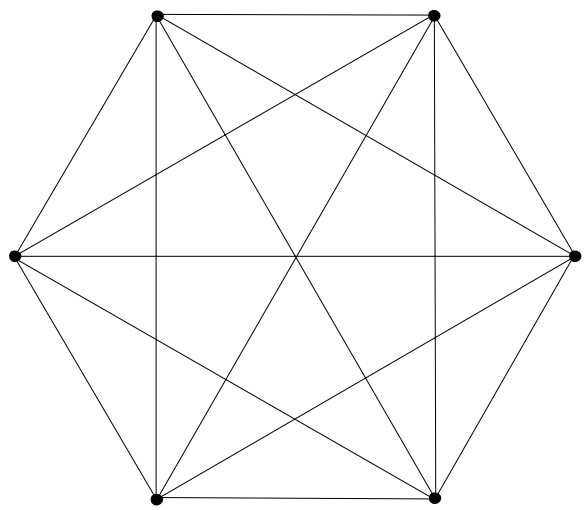

Figure 6: **Complete Graph on 6 Nodes**

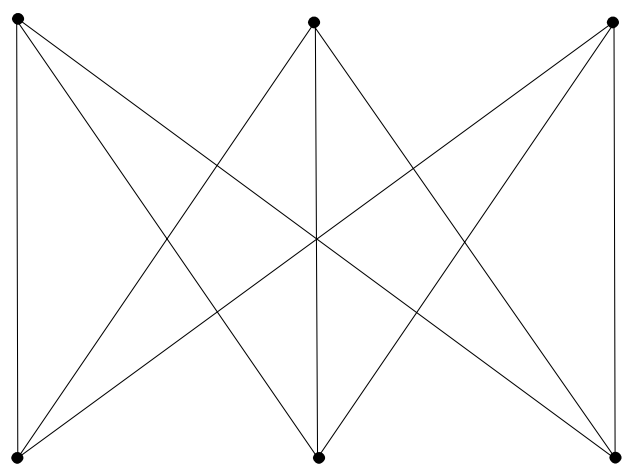

Figure 7: **A Bipartite Graph on 6 Nodes**

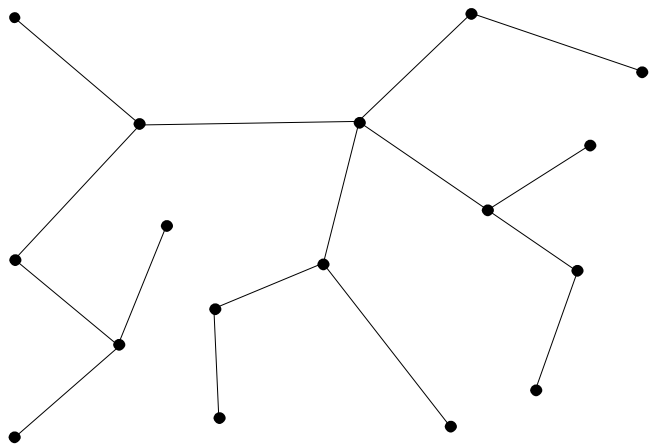

Figure 8: **A Tree Graph**

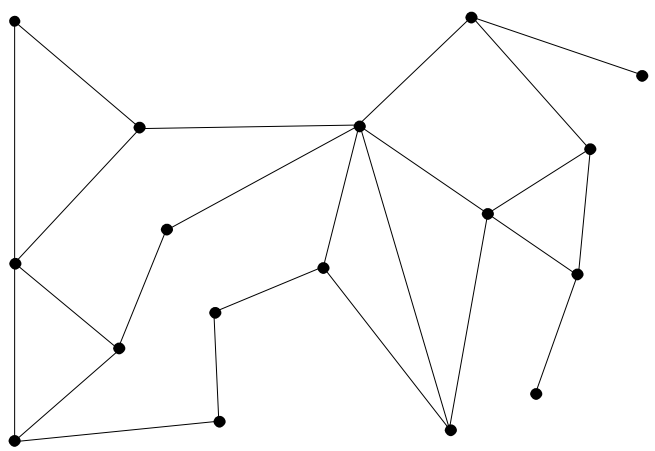

Figure 9: **An Arbitrary Graph**