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Global Economic Growth**

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Abstract

This paper develops a fully-endogenous, variety-expansion growth model with firm-specific quality heterogeneity, limit pricing, and an endogenous distribution of markups. Trade induces only firms with high-quality products to export, whereas firms with low-quality products serve only the domestic market. Trade liberalization, measured by a reduction in trade costs or a reduction in foreign-market entry costs, shifts resources from low-quality to high-quality products and intensifies the product market competition. However, it has ambiguous effects on the average markup, long-run growth, and welfare. An increase in the rate of population growth or in the intensity of international knowledge spillovers accelerates economic growth. The laissez-faire equilibrium is inefficient, and this leaves room for welfare-improving government intervention.

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1 Introduction

Several firm-level empirical studies have convincingly argued that the existence of large and persistent productivity and/or product quality differences among firms within narrowly defined product categories accounts for observed trade patterns in monopolistically competitive markets.¹ These studies have shown that more productive (high-quality product) firms charge lower (higher) prices and are more likely to enter foreign markets after paying large foreign-market entry costs.

The present paper analyzes the long-run effects of trade liberalization in the presence of quality heterogeneity, market-entry costs, and trade costs. We consider a global economy consisting of two structurally identical countries where new varieties are discovered through resource-using R&D investments. Labor is the only factor of production and the supply of labor grows at a constant growth rate. We assume that the productivity of researches depends positively on domestic and foreign knowledge spillovers as in Rivera-Batiz and Romer (1991), and negatively on market size measured by the level of population. These two assumptions deliver fully endogenous growth which depends on virtually all parameters including those that capture the effects of trade liberalization.

The R&D process, which uses labor and available knowledge, is stochastic in the sense that a firm must incur an instantaneous fixed cost in order to draw its product quality from a known distribution.² Therefore, firms produce products with different quality levels. Each surviving firm faces a unit-elastic demand curve and competes against a competitive fringe of imitators producing a generic low-quality product in a Bertrand fashion. As in quality-ladders growth models, firms optimally charge a limit price, which is proportional to the quality of their products and drive the competitive fringe out of the market. Consequently, firms with higher-quality products charge higher prices and enjoy higher markups

¹See Clerides et al. (1998), Bernard and Jensen (1999), Aw et al. (2000), Schott (2004), Hummels and Skiba (2004), Hummels and Klenow (2005), and Hallak (2006) among many others.

²The product innovation process is similar to that in Melitz (2003) and Dinopoulos and Unel (2009). However, their models assume that firms fully internalize knowledge spillovers, i.e., firms can not benefit from technologies developed by other producers. As a result, the growth of varieties is zero.

and profits. Exporting requires additional costs that take the form of an instantaneous fixed cost and a per-unit trade cost. Thus only firms producing high-quality products engage in exporting. In contrast, firms with low-quality products serve the domestic market only. Since entry into the domestic market requires an instantaneous fixed cost, firms that discover goods of very low quality exit the market.

Having developed a fully-endogenous growth model with heterogeneous firms, we then analyze the determinants of long-run growth and welfare. The presence of quality heterogeneity introduces two novel features in the theory of endogenous growth. First, quality uncertainty implies that the effective instantaneous fixed cost of discovering a new surviving variety is endogenous and depends on trade and foreign-market entry costs in an ambiguous way. In contrast, in growth models with symmetric firms, each firm faces a constant instantaneous fixed cost of discovering a new variety. Second, the presence of an endogenous distribution of quality levels implies that market profitability, measured by average markup, is also endogenous. This introduces another endogenous component of long-run growth which is absent from traditional growth models. An increase in expected instantaneous fixed costs weakens the incentives to innovate, whereas an increase in the average markup strengthens the incentives to innovate. In our model, we show that trade liberalization has ambiguous effect on the average markup and expected instantaneous fixed costs, and consequently, has an ambiguous effect on long-run growth. In contrast, an increase in the intensity of foreign knowledge spillovers raises long run growth. Finally, we establish that the laissez-faire equilibrium is inefficient, and this leaves room for welfare-improving government intervention.

The present paper is related to a growing literature that investigates the effects of trade on growth in the presence of heterogeneous firms. Baldwin and Robert-Nicoud (2008) and Unel (2008) generalize Melitz's (2003) model by introducing a growth mechanism based on earlier endogenous growth models. They find that exposure to trade has an ambiguous effect on long-run growth. However, their model generates constant average markups and

therefore does not taking into account the second type of pro-growth effect of trade liberalization described above. Moreover, their model suffers from the scale-effects property in the sense that the growth rate is proportional to the population size. Gustafsson and Segerstrom (2007) develop a model of semi-endogenous growth and productivity heterogeneity in which the long-run growth rate of varieties is directly proportional to the exogenous rate of population growth only. It also generates constant markups due to the use of Dixit and Stiglitz (1977) preferences. In contrast, the present paper generates fully-endogenous global growth of varieties, which depends (positively) on the rate of population growth, and is affected by several other parameters of interest including those that capture the degree of trade liberalization.

In another related study Haruyama and Zhao (2008) introduce firm-specific productivity heterogeneity in a quality ladders growth model. In their model, if the R&D process generates endogenous growth (with scale effects), trade liberalization unambiguously raises the long-run growth rate of total factor productivity; whereas under a scale-invariant R&D process, their model generates semi-endogenous long-run growth which is not affected by trade liberalization. Our paper differs in many dimensions from Haruyama and Zhao (2008) including the growth mechanism, and complements their main finding by establishing that trade liberalization has an ambiguous effect on growth when the latter is generated by variety accumulation (as opposed to quality improvements).

The rest of the paper is organized as follows. Section 2 introduces the elements of the model and establishes the uniqueness of the steady-state equilibrium. Section 3 analyzes the long-run effects of trade liberalization. Section 4 investigates the welfare implications of the model in a closed-economy set up. Section 5 offers some concluding remarks.

2 The Model

We consider a global economy consisting of two structurally identical countries. Each country is populated by a continuum of firms whose products have different quality levels. Labor

is the only factor of production and grows at a constant rate. The creation of new varieties follows the R&D process similar to Melitz (2003): the quality level of each discovered variety is drawn from a probability distribution with positive support after each entrant incurs an instantaneous fixed R&D cost. In addition, each firm faces fixed instantaneous domestic-market entry costs, fixed instantaneous foreign-market-entry costs, and fixed per-unit trade costs .

2.1 Consumer Behavior

In each country there is a unit measure of identical households. The representative household is modeled as a dynastic family that grows over time at the constant rate of population growth $n > 0$. Normalizing the initial number of family members to one, the population level (and each family's size) at time t is $L_t = e^{nt}$. The preferences of the representative household are given by the following intertemporal utility function

$$U = \int_0^{\infty} e^{-(\rho-n)t} \ln u_t dt, \quad (1)$$

where $\rho - n$ denotes the effective discount rate and $\ln u_t$ denotes each consumer's instantaneous utility function. The latter is given by the following Cobb-Douglas function over a continuum of goods indexed by ω

$$\ln u_t = M_{ct} \ln M_{ct} + \int_0^{M_{ct}} \ln \left[\lambda(\omega) \frac{q_t(\omega)}{L_t} \right] d\omega, \quad (2)$$

where $\lambda(\omega)$ denotes the *time-invariant* product quality, $q(\omega)$ is the aggregate consumption of brand ω , and M_{ct} denotes the mass of varieties available for consumption. The first term in (2) directly captures the individual's love of varieties and its inclusion into the utility function is necessitated to ensure that consumers demand all available products in the market.³

³The condition $f_x > 2/(\rho - n)$, where f_x is instantaneous fixed foreign market entry cost, guarantees that the consumption non-satiation principle holds and that consumers buy all available varieties. Section 4 provides more details on the derivation of this condition.

The household maximization problem can be solved in two steps. First, each consumer maximizes the instantaneous utility function (2) subject to the budget constraint $\int_0^{M_{ct}} p_t(\omega)q_t(\omega)/L_t d\omega \leq E_t$, where $p(\omega)$ is the corresponding price of the brand and E_t is the *per-capita* consumption expenditure. The solution to this maximization problem yields the following expression for the market demand for a typical variety

$$q_t(\omega) = \frac{E_t L_t}{p_t(\omega) M_{ct}}. \quad (3)$$

This optimal consumption rule implies that the aggregate demand for a product increases in aggregate consumer expenditure $E_t L_t$; and decreases in its own price $p_t(\omega)$, and the number of available products M_{ct} .

Second, each consumer maximize (1) subject to the intertemporal budget constraint

$$\int_0^{\infty} e^{-R_t} E_t dt \leq \mathcal{A}_0,$$

where R_t is the cumulative discount factor and \mathcal{A}_0 is the present value of the household wealth. The optimal spending path is given by the following Euler equation

$$\frac{\dot{E}_t}{E_t} - \frac{\dot{M}_{ct}}{M_{ct}} = r_t - \rho, \quad (4)$$

where r_t is the instantaneous interest rate. In our subsequent analysis, we will drop the time index to simplify the notation, and we will do so when this causes no notational confusion.

2.2 Product Markets

There is a continuum of firms, each choosing to produce a different variety, and we assume that production of one unit of each good requires one units of labor. Firms wishing to export must incur per-unit trade costs. Iceberg trade costs (such as transport costs and tariffs) are modeled in the standard fashion: $\tau > 1$ units of output must be produced at home in order for one unit to arrive at its destination.

The aggregate quantity demanded, which is given by equation (2), implies that expenditure per variety is identical across varieties and independent of product quality. Since

the elasticity of demand for each variety is unity, firms have incentives to charge an infinite price and produce almost zero quantity independently of the quality level. To prevent this from happening, we assume that once a product is consumed in a market, a generic, lower-quality version of the product can be produced by a competitive fringe. The production of generic products is characterized by constant returns to scale where one unit of labor produces one unit of output. We assume that the generic version of a product cannot be produced in a country unless the original product is sold there.⁴ We normalize the quality level of each generic good to unity independently of the quality level of the copied product and the location of production.

Since each brand is associated with a unique quality level, hereafter we shall label products based on their quality level. Let $p_d(\lambda)$ and $p_x(\lambda)$ denote the consumer price prevailing in the domestic and foreign markets respectively, and assume that competition within each product occurs in a Bertrand fashion. The possibility of costless imitation forces firms to maximize profits by charging a (limit) price no higher than $p_d(\lambda) = p_x(\lambda) = w\lambda$, where w is the common wage rate across both countries, hereafter normalized to unity, that is $w = 1$. The price competition drives domestic and foreign imitators out of the market and ensures that firms with higher-quality products charge higher prices.⁵

The limit-pricing rule and (3) yield

$$\frac{q(\lambda_1)}{q(\lambda_2)} = \frac{p(\lambda_1)}{p(\lambda_2)} = \frac{\lambda_2}{\lambda_1}. \quad (5)$$

That is, firms with higher-quality products charge higher prices and sell lower quantities. However, all firms earn the same revenue $p(\lambda)q(\lambda)$ as in the standard quality-ladders growth model.

⁴Alternatively, we can assume technology diffuses instantly across countries: once a product is developed, its low-quality generic version can be produced by a competitive fringe in any country. However, analysis based on this assumption yields qualitatively similar results, and is available upon request.

⁵Segerstrom et al. (1990) and Grossman and Helpman (1991, Chapter 4) and Aghion and Howitt (1992), for example, provide excellent discussion on how such copying technology can generate the aforementioned optimal limit-pricing rule.

Under this limit-pricing rule, a firm's flows of domestic and export profits $\pi_d(\lambda)$ and $\pi_x(\lambda)$, respectively, are given by

$$\pi_d(\lambda) = [p_d(\lambda) - 1]q_d(\lambda) = (1 - \lambda^{-1})EL/M_c, \quad (6)$$

$$\pi_x(\lambda) = [p_x(\lambda) - \tau]q_x(\lambda) = (1 - \tau\lambda^{-1})EL/M_c, \quad (7)$$

where the quantities demanded by domestic and foreign consumers $q_d(\lambda)$ and $q_x(\lambda)$ are given by (3) and $p_d(\lambda) = p_x(\lambda)$.

The decision to discover a new variety depends on the comparison between the expected instantaneous fixed costs to introduce, produce and export a product and the expected discounted benefits associated with the profit flows defined by (6) and (7). The net discounted value of profits at time t (denoted by ν_{it} , for $i = d, x$) are equal to

$$\nu_{it}(\lambda) = \int_t^\infty e^{-[R_s - R_t]} \pi_{is}(\lambda) ds, \quad (8)$$

where R_t is the cumulative interest factor from time 0 to time t .

Differentiating (8) with respect to time delivers a more intuitive expression that defines the discounted value of profits ν_{it} :

$$\pi_{it}(\lambda) + \dot{\nu}_{it}(\lambda) = r_t \nu_{it}(\lambda). \quad i = d, x.$$

The left-hand-side equals the return to equity in a firm with quality level λ : owners of this firm earn the flow of profits $\pi_{it}dt$ during the infinitesimal interval of time dt and the capital gain $d\nu = \dot{\nu}_{it}dt$. Because there is no risk of default for a surviving product once its quality is known, the total return to firm's equity must be equal to the flow of money generated by an equal-size investment earning the market interest rate $r_t \nu_{it}dt$. Solving the above expression for the discounted value of profits yields

$$\nu_{it}(\lambda) = \frac{\pi_{it}(\lambda)}{r_t - \dot{\nu}_{it}/\nu_{it}}. \quad (9)$$

The discounted value of profits earned by a surviving firm equals the flow of profits π_{it} discounted by the market interest rate r_t minus the growth rate of ν_{it} . In the absence of

population growth, $\dot{\nu}_{it} = 0$ in the long-run, and (9) yields $\nu_{it} = \pi_{it}(\lambda)/r_t$ as in Romer (1990).⁶

2.3 Entry and Exit Decision

The presence of substantial market-entry costs has been documented by several empirical studies (e.g., Roberts and Tybout (1997), Bernard and Jensen (2004)). In addition, Romer (1994, p24) offers the following example for the type of fixed costs associated with exporting: “One can think, for example, of the information a foreign retailer must collect about quality, reliability, and capacity of suppliers before it can begin to buy garments assembled in a new country. The retailer would have to establish new financial relationships for clearing transactions and new shipping and communication links for moving goods. It would have to learn about the local legal, regulatory, and tax environment and it would have to investigate the nature of political risk. It would also need to invest in long term implicit and explicit contractual relationships for the trading relationship to be successful.”

In our model, a firm has to decide whether or not to discover a blueprint and where to sell its product. This decision is based on three distinct instantaneous sunk costs: R&D costs $f_e\phi_t$ to allow the firm to learn its quality level and create a blueprint; R&D costs $f_d\phi_t$ to learn how to produce and sell its product in the domestic market; and additional R&D costs $f_x\phi_t$ to learn how to sell the good in the foreign market. These three sunk costs can be interpreted as different unit labor requirements since labor is used as the numeraire ($w = 1$). Individual firms treat these sunk costs as parameters, but they can change over time due to changes in the size of the market measured by the level of population or due to international knowledge spillovers.

The first component of each type of sunk costs (f_e , f_d , or f_x) is a fixed parameter as in Melitz (2003). The common component ϕ_t , which governs the evolution of fixed costs over time and delivers fully-endogenous growth, is defined as

⁶ Notice that, in contrast to Melitz (2003) where firms face an exogenous risk of default in each period, firms in the present model do not face any default risk. One can introduce an exogenous risk of default following the endogenous growth literature without altering the results of the analysis.

$$\phi_t = \frac{L_t}{(1 + \theta)M_{dt}} \quad (10)$$

where L_t is each country's labor force, parameter $\theta \in [0, 1]$ measures the degree of foreign knowledge spillovers, and M_{dt} is the measure of varieties produced in each country by the domestic producers.

The function ϕ_t reveals two competing economic forces that shape the magnitude of instantaneous fixed costs. First, the numerator of (10) captures the notion that firms in larger markets face higher fixed costs. These fixed costs make the learning process more difficult and capture the presence of scale diseconomies. Diseconomies of scale can arise from higher costs of setting up distribution systems in larger markets; larger advertising expenditures; the spreading of talent and other specialized resources too thin in larger markets; higher costs associated with coordination failures and costs associated with the flow of information. Following the fully-endogenous growth literature, we assume that the difficulty of learning is proportional to the size of each country's market measured by its level of population.⁷ The assumption that ϕ_t is linear in L_t is consistent with several fully endogenous growth models (see, for instance, Young, 1998; Dinopoulos and Thompson, 1998; Dinopoulos and Segerstrom, 1999; and Howitt, 1999).

The denominator of (10) captures the notion that the stock of knowledge, which can be modeled as being proportional to the mass of varieties available at time t , enhances research productivity and reduces the learning costs, as in Romer (1990) and Rivera-Batiz and Romer (1991). In a trading world, it is reasonable to assume that the foreign contribution to local knowledge increases with the volume of trade. For example, local researchers can study the embodied knowledge in imported goods or the distribution systems that importers build to market and sell their products. Also, exposure to international trade could increase the number of contracts between domestic and foreign producers, which then could expand the flow of knowledge between countries (see Grossman and Helpman, 1991; and Coe and

⁷The results of the paper are invariant to measuring the market size by the level of world population $2L_t$ instead of the level of each county's population level L_t .

Helpman, 1995).⁸ Therefore, we assume that the stock of knowledge in each country equals the measure of varieties produced locally M_{dt} plus the effective measure of varieties produced abroad θM_{dt} , which equals the denominator of (10). We assume that θ increases with the volume of trade,⁹ and to simplify notation, we further assume that knowledge spillovers are transferred only through trade, i.e., $\theta = 0$ in autarky.

Each variety is associated with a quality level λ , which is randomly drawn from a common distribution $g(\lambda)$, and an associated cumulative distribution $G(\lambda)$. Introduction of a new product in each market, however, requires the innovator to pay additional sunk costs of adapting the variety to the market standards and regulations. Upon developing a new product, the innovator checks the associated quality level λ . If the firm's quality level is low such that the discounted sum of profits earned from local sales is less than the domestic-market entry cost $f_d\phi_t$, then the firm exits the market. If the firm's discounted sum of profits earned from local sales is greater than $f_d\phi_t$, then it will bear these once-for-all sunk costs and the associated flow of variable costs in order to enter successfully the domestic market. If the quality level is high enough to also cover the foreign-market-entry costs $f_x\phi_t$, then the innovator incurs $(f_d + f_x)\phi_t$ fixed costs at time t in order to serve both the local and foreign markets. Consequently, unlike Melitz's (2003) model where firms incur fixed domestic and export costs in each period of time and face an exogenous probability of default in each period, in the present model infinitely-lived firms face once-and-for-all (instantaneous) fixed costs. These assumptions are not crucial for our results, but deliver more tractability and follow more closely the endogenous growth theory.

Inspection of (6) and (7) reveals that $\pi_d(\lambda)$ and $\pi_x(\lambda)$ increase monotonically in the product quality level λ . Consequently, ν_d and ν_x are also increasing in λ . Furthermore, equations (6) and (7) imply that $\pi_d(1) = \pi_x(\tau) = 0$, which in turn ensure that $\nu_d(1) =$

⁸A large empirical literature investigates the nature and magnitude of trade-related technology spillovers. These studies argue that trade is an important conduit for international technology transfer. Keller (2004) offers an insightful review of this strand of literature.

⁹Alternatively, it can be assumed that the fraction of technology transferred from the other country is an increasing function of the fraction of foreign firms that export to home (see Baldwin and Robert-Nicoud, 2008). Analysis based on this specification doesn't change the results.

$\nu_x(\tau) = 0$. Thus, among the observed quality levels, there exist two cutoff quality levels λ_d and λ_x such that

$$\nu_d(\lambda_d) = f_d \phi_t, \quad (11)$$

$$\nu_x(\lambda_x) = f_x \phi_t. \quad (12)$$

These equations define the domestic and export cutoff quality levels λ_d and λ_x . Following the literature on heterogeneous firms, we will later impose sufficient and reasonable conditions to guarantee that the domestic cutoff quality level is strictly less than the export cutoff quality level (see Proposition 1). Consequently, firms in each country are partitioned in three distinct groups. Firms that discover products with quality $\lambda < \lambda_d$ can not profitably serve even the local market and exit. Firms that discover intermediate-quality products, such that $\lambda_d \leq \lambda < \lambda_x$, serve only the local (domestic) market. Consequently, only firms that discover products with a quality level $\lambda > \lambda_x$ serve profitably the foreign market.

The above considerations imply that the ex-post distribution of surviving product quality levels $\mu(\lambda)$ is the conditional distribution of $g(\lambda)$ on the interval $[\lambda_d, \infty)$

$$\mu(\lambda) = \begin{cases} \frac{g(\lambda)}{1-G(\lambda_d)} & \text{if } \lambda > \lambda_d \\ 0 & \text{otherwise} \end{cases} \quad (13)$$

And the ex-ante probability that one of these successful firms will export is given by

$$\zeta_x = \frac{1 - G(\lambda_x)}{1 - G(\lambda_d)}. \quad (14)$$

The law of large numbers implies that ζ_x also equals the ex-post fraction of incumbent firms that export. Thus, the measure of firms who successfully enter into export market is $M_x = \zeta_x M_d$. Structural symmetry across the two countries implies that the mass of varieties available for consumption in each market is given by $M_c = (1 + \zeta_x)M$. Based on these cutoff quality levels, the value of a surviving firm with product quality λ is given by

$$\nu(\lambda) = \begin{cases} \nu_d(\lambda) & \text{if } \lambda_d \leq \lambda \leq \lambda_x \\ \nu_d(\lambda) + \nu_x(\lambda) & \text{if } \lambda_x \leq \lambda. \end{cases} \quad (15)$$

The model generates ex-post quality heterogeneity which delivers heterogeneous limit prices, heterogeneous quantities produced, heterogeneous markups, and heterogeneous discounted profits (according to (15)). However, firms are ex-ante identical and the incentives to create new varieties, which in turn fuel economic growth, are based on ex-ante calculations that involve a comparison of expected costs to expected discounted benefits. We assume that there is free-entry into the creation of new varieties which drives the expected discounted profits down to zero. Armed with the product survival probability density $\mu(\lambda)$, given by (13), we state the innovation free-entry condition as

$$[1 - G(\lambda_d)] \left[\int_{\lambda_d}^{\infty} \nu(\lambda) \mu(\lambda) d\lambda - (f_d \phi + \zeta_x f_x \phi) \right] = f_e \phi, \quad (16)$$

where the time index has been omitted from functions and variables. If a firm draws a quality level higher than or equal to λ_d , it earns a discounted stream of profits $\bar{\nu} = \int_{\lambda_d}^{\infty} \nu(\lambda) \mu(\lambda) d\lambda$ minus the expected instantaneous costs of entering the local and foreign markets $f_d \phi + \zeta_x f_x \phi$; therefore, the expected discounted benefits of drawing a “winning variety” with quality level $\lambda \geq \lambda_d$ and serving the local, or both the local and foreign markets, is given by the left-hand-side of (16). The value of net expected discounted benefits of producing and marketing a product must be equal the costs of discovering the product, which is equal to the right-hand side of (16).

Performing the integration and collecting terms one can rewrite the above free-entry condition (16) as

$$\int_{\lambda_d}^{\infty} \nu(\lambda) \mu(\lambda) d\lambda = \bar{f} \phi, \quad (17)$$

where the left-left hand side of (17) captures the expected discounted benefits of creating a surviving product and the right-hand-side equals the expected instantaneous fixed costs of discovering a new variety and successfully entering the local and foreign markets. The term \bar{f} is instantaneous component of these expected fixed costs defined by

$$\bar{f} = \frac{f_e}{1 - G(\lambda_d)} + f_d + \zeta_x f_x. \quad (18)$$

The first term in (18) captures the expected costs to develop a successful variety f_e times the inverse of the probability of drawing a product with quality higher or equal to the production cutoff quality level λ_d , which equals the expected number of attempts at developing a surviving product. The second term represents the instantaneous cost of entering the local market. The third term is the expected instantaneous costs of becoming a successful exporter, which is proportional to $\zeta_x f_x$, where the probability of serving the foreign market (ζ_x) is given by (14).

2.4 Evolution of Varieties

The economy-wide flow of new varieties is determined as follows. Denote with L_e the aggregate (economy-wide) amount of labor devoted to the discovery of new varieties. The discovery of a surviving single variety requires $(1/\phi\bar{f})dt$ labor costs, where dt denotes an infinitesimal interval of time. Therefore the aggregate flow of varieties produced in each economy is given by $dM_d = (L_e/\phi\bar{f})dt$ which can be written as $\dot{M}_d = L_e/\phi\bar{f}$. Substituting the value of ϕ from (10) in this expression yields the growth rate of varieties

$$\gamma \equiv \frac{\dot{M}_d}{M_d} = \frac{(1+\theta)L_e}{\bar{f}L}. \quad (19)$$

According to equation (19), the growth rate of varieties increases in the degree of international knowledge spillovers θ and in the number to researchers devoted to innovative R&D L_e ; and it decreases in the effective expected fixed cost per variety \bar{f} , defined by (18), and the R&D difficulty, measured by market size L .

A few remarks on the interpretation of (19) and its relation to the rest of the literature are in order. First, Melitz (2003) and Dinopoulos and Unel (2009) among many others abstract from long-run growth considerations and analyze a long-run equilibrium where the mass of varieties is constant over time, that is $\gamma = 0$. Second, Romer (1990) and Rivera-Batiz and Romer (1991) assume that the R&D difficulty is constant over time, i.e., $L_t = \bar{L}$ where \bar{L} is a parameter. This restriction introduces the undesirable scale-effects property which implies that the long-run growth of varieties is unbounded in the presence of positive

population growth. Jones (1995a) has convincingly argued that the scale-effects property of earlier endogenous growth models is inconsistent with time-series evidence from several advanced countries. To remove the scale effects-property from our model, we assume that the development of a new variety depends on the average stock of knowledge $(1 + \theta)M_d/L_t$, rather than the aggregate stock of knowledge $(1 + \theta)M_d$. This specification of R&D difficulty delivers fully-endogenous growth differs from semi-endogenous growth models, which assume that R&D difficulty is proportional to $[(1 + \theta)M_d]^{1-\psi}$ with $\psi < 1$, and deliver exogenous long-run growth that depends only on parameter ψ and the rate of population growth n (Jones, 1995b; Kortum, 1997; and Segerstrom, 1998).¹⁰

The focus of the present model on fully-endogenous growth differentiates our approach from several related studies that analyze the nexus between growth and trade in the presence of heterogeneous firms. For example, Baldwin and Robert-Nicoud (2008) analyze the effects of productivity heterogeneity on long-run growth using a first-generation model of endogenous growth with scale effects. As such, their model abstracts from issues related to population growth. Gustafsson and Segerstrom (2007) analyze the effects of trade liberalization in a model with productivity heterogeneity and semi-endogenous growth, where trade does not affect long-run growth but has an ambiguous impact on welfare. Haruyama and Zhao (2008) develop first generation and semi-endogenous quality ladders growth models to study the effects of trade liberalization on the level of productivity. Our model is the first fully endogenous growth model of quality-heterogeneity and trade, and as such it complements these studies. It is apparent from (18) and (19) that, unlike other related studies, there are two channels that transmit the effects of trade liberalization on long-run growth: the expected fixed R&D costs \bar{f} which depend on the cutoff quality levels λ_d and

¹⁰Our specification is also similar to Lucas's (1988) human capital model, in which human capital accumulates through learning and spillovers; and each producer benefits from the average human capital. Furthermore, Ha and Howitt (2007) argue that long-run trends in R&D and TFP are more supportive of fully-endogenous growth theory than they are of the semi-endogenous growth theory proposed by Jones (1995b). Nevertheless, we also analyzed a semi-endogenous growth version of our paper and found that exposure to trade does not affect the long-run growth rate, but has an ambiguous level effect on welfare. These results are qualitatively similar to those in Gustafsson and Segerstrom (2007), and are available upon request.

λ_x ; and the share of labor devoted to R&D L_e/L . These two channels transmit the effects of virtually all parameters on long-run growth including changes in the rate of population growth that are missing from first-generation endogenous growth models.

2.5 Aggregation and Labor Markets

The next step of the analysis is to derive the full-employment of labor condition in each country. The assumption of structural symmetry between the two countries suffices to analyze only the Home economy. To this end, we first represent key variables in a more compact way. The aggregate quantity demanded can be written as

$$Q = \int_{\lambda_d}^{\infty} q(\lambda)M_d\mu(\lambda)d\lambda + \int_{\lambda_x}^{\infty} q(\lambda)M_d\mu(\lambda)d\lambda = \frac{M_d}{M_c} \frac{EL}{\tilde{\lambda}_d} + \frac{M_x}{M_c} \frac{EL}{\tilde{\lambda}_x}, \quad (20)$$

where λ_i ($i = d, x$) is given by

$$\tilde{\lambda}_i \equiv \tilde{\lambda}(\lambda_i) = \left[\frac{1}{1 - G(\lambda_i)} \int_{\lambda_i}^{\infty} \lambda^{-1} g(\lambda) d\lambda \right]^{-1}. \quad (21)$$

Thus, $\tilde{\lambda}_d$ is the weighted harmonic mean of the quality levels of all produced goods and can be interpreted as the average quality level. Similarly, $\tilde{\lambda}_x$ is the weighted of the harmonic mean of the quality levels of a country's exports and can be interpreted as the average export quality.

As also noted by Dinopoulos and Unel (2008), since λ_i is the minimum quality level of all products with $\lambda \in [\lambda_i, \infty)$, it must be lower than the average quality level of all products within this quality range, i.e., $\lambda_i < \tilde{\lambda}_i$. Moreover, (18) indicates that an increase in λ_i forces producers with low-quality products to exit the market. This in turn increases the corresponding average quality level, i.e., $\tilde{\lambda}_i$. These properties are summarized below.

$$\lambda_i < \tilde{\lambda}_i; \quad \partial \tilde{\lambda}_i / \partial \lambda_i > 0, \quad \text{for } i = d, x. \quad (22)$$

We assume that labor markets are perfectly competitive, and therefore labor is fully employed in each instant in time. The aggregate supply of labor is L , whereas the aggregate demand for labor comes from fixed costs associated with different R&D activities and

manufacturing of final consumption goods. Labor devoted to effective R&D, which includes instantaneous fixed market-entry costs, is given by $L_e = \gamma \bar{f} L / (1 + \theta)$ (see equation (20)). The demand for labor employed in manufacturing is derived as follows. One unit of labor produces one unit of output. Consequently, each surviving firm serving its local market hires $q(\lambda)$ workers; and each firm serving the export market hires $\tau q(\lambda)$ workers (in order to deliver $q(\lambda)$ units abroad), where $q(\lambda) = EL / \lambda M_c$ is the quantity demanded. Therefore the aggregate quantity of labor demanded in each country is given by

$$L_p = \int_{\lambda_d}^{\infty} q(\lambda) M_d \mu(\lambda) d\lambda + \int_{\lambda_x}^{\infty} \tau q(\lambda) M_d \mu(\lambda) d\lambda = \frac{M_d EL}{M_c \tilde{\lambda}_d} + \tau \frac{M_x EL}{M_c \tilde{\lambda}_x} = \frac{EL}{\tilde{\lambda}}, \quad (23)$$

where $\tilde{\lambda}$ is the (distance-adjusted) weighted average quality of all products competing in each market, and it is given by

$$\tilde{\lambda} = \left\{ \frac{1}{M_c} [M_d \tilde{\lambda}_d^{-1} + \tau M_x \tilde{\lambda}_x^{-1}] \right\}^{-1} = [1 + \zeta_x] [\tilde{\lambda}_d^{-1} + \tau \tilde{\lambda}_x^{-1}]^{-1}, \quad (24)$$

where ζ_x , defined by (14), is the ex-post fraction of varieties exported (and imported). Variable $\tilde{\lambda}$ plays key role in our subsequent analysis, and as shall be shown shortly, the average markup across all incumbents in each economy is given by $1 - \tilde{\lambda}^{-1}$.

In view of (23), the full-employment of labor condition $L_e + L_p = L$ can be written as

$$\frac{\gamma \bar{f}}{1 + \theta} + \frac{E}{\tilde{\lambda}} = 1, \quad (25)$$

where \bar{f} and $\tilde{\lambda}$ are defined by (18) and (24), respectively. This completes the description of the model.

2.6 Steady-State Equilibrium

This section describes the basic properties of steady-state (balanced-growth) equilibrium, where all endogenous variables grow at constant, but not necessary equal, rates over time. We start by requiring that the steady-state variety growth rate $\gamma = \dot{M}_d / M_d$ must be constant over time. In addition, observe that the fraction $\zeta_x = [1 - G(\lambda_x)] / [1 - G(\lambda_d)]$, which is

the share of exported varieties, must be constant over time. Otherwise the economy ends up with all varieties exported or no trade. These two requirements imply that the steady state growth of exported and consumed varieties is constant over time, i.e., $\dot{M}_c/M_c = \dot{M}_x/M_x = \gamma$. They also imply that the following variables are constant over time: the two cutoff quality levels λ_d and λ_x ; the average quality levels $\tilde{\lambda}_d$, $\tilde{\lambda}_d$, and $\tilde{\lambda}$; and the expected instantaneous fixed costs \bar{f} . Taking all this into account, observe that the full employment of labor condition (25) implies that per-capita expenditure must be constant over time.

Since the cutoff quality levels λ_d and λ_x are time invariant, one can derive the following steady-state expressions¹¹ for discounted profits earned by a surviving firm producing a product of quality λ :

$$\nu_d(\lambda) = \frac{\pi_d(\lambda)}{\rho - n} = (1 - \lambda^{-1}) \frac{EL}{M_c}, \quad (26)$$

$$\nu_x(\lambda) = \frac{\pi_x(\lambda)}{\rho - n} = (1 - \tau \lambda^{-1}) \frac{EL}{M_c}, \quad (27)$$

where the profit flows π_d and π_x , defined by (6) and (7) were substituted as well. Substituting (26) and (27) into the zero-profit conditions (11) and (12), respectively, and dividing the two resulting equations yields

$$\lambda_x = \frac{\tau}{1 - (1 - \lambda_d^{-1})f_x/f_d}. \quad (28)$$

Equation (28) shows the exporting cutoff quality level λ_x as a function of the domestic cutoff quality level λ_d and the model's parameters. Several remarks are in order. First, the denominator of (28) must be non-negative, and therefore we must have that $\lambda_d \in (1, k)$, where $k = 1/[1 - f_d/f_x]$. Second, according to (28), λ_x is an increasing function of the domestic cutoff quality level λ_d the transportation costs τ and the ratio of overhead fixed costs f_x/f_d . Finally, (28) indicates that when $f_x \geq f_d$ and $\tau > 1$, the export cutoff quality level λ_x is strictly greater than the domestic cutoff quality level λ_d .

¹¹Substituting (10) into (11) and (12), and differentiating the resulting expression with respect to time yields $\dot{\nu}_i/\nu_i = n - \gamma$. In addition, equation (4) implies $r_t = \rho - \gamma$, since per-capita expenditure is time invariant. Inserting these two expressions in the denominator of (9) yields (26) and (27).

The next step is to solve for the steady-state value of the domestic cutoff quality level.

The innovation free-entry condition (17) can be written as

$$\frac{1}{1 - G(\lambda_d)} \int_{\lambda_d}^{\infty} \nu_d(\lambda) g(\lambda) d\lambda + \frac{\zeta_x}{1 - G(\lambda_d)} \int_{\lambda_x}^{\infty} \nu_x(\lambda) g(\lambda) d\lambda = \bar{f}\phi, \quad (29)$$

and can further be expressed as¹²

$$f_d H(\lambda_d, 1) + f_x H(\lambda_x, \tau) = f_e, \quad (30)$$

where $H(\lambda_i, \alpha)$ (for $i = d, x$ and $\alpha = 1, \tau$, respectively) is defined as follows:

$$H(\lambda_i, \alpha) = [1 - G(\lambda_i)] \left[\frac{1 - \alpha \tilde{\lambda}_i^{-1}}{1 - \alpha \lambda_i^{-1}} - 1 \right]. \quad (31)$$

Equations (28) and (30) constitute a system of two equations in two unknowns, the cutoff quality levels λ_d and λ_x . The function $H(\lambda_i, \cdot)$ is strictly decreasing in λ_i .¹³ In addition, λ_x is an increasing function of λ_d , and therefore these two equations yield a unique solution for (λ_d, λ_x) . The following proposition summarizes these results.

Proposition 1. *Let $k = 1/(1 - f_d/f_x)$ and assume that $f_d < f_x$. In the steady-state equilibrium, there exist **unique** and **time-invariant** domestic and export cutoff levels λ_d and λ_x which satisfy equations (28) and (30) such that $\lambda_d > 1$, $\lambda_x > \tau$, and $\lambda_x > \lambda_d$.*

Having determined the steady-state values of domestic and export cutoff levels, one can easily determine the values of several endogenous variables. Equation (14) pins down the time-invariant endogenous fraction of traded varieties ζ_x . Another variable of interest is the market profitability. Using the profit flows (6) and (7), one can write the aggregate profits of all surviving firms in each country as

$$\Pi = \int_{\lambda_d}^{\infty} \pi_d(\lambda) M_d \mu(\lambda) d\lambda + \int_{\lambda_x}^{\infty} \pi_x(\lambda) M_d \mu(\lambda) d\lambda = (1 - \tilde{\lambda}^{-1}) EL, \quad (32)$$

¹²Substituting (26) and (27) (evaluated at the cutoff quality levels) into (11) and (12), respectively, yields $EL/M_c = (\rho - n)f_d\phi/[1 - \lambda_d^{-1}]$ and $EL/M_c = (\rho - n)f_x\phi/[1 - \tau\lambda_x^{-1}]$. Inserting these expressions back into (26) and (27), and the resulting expressions into (29), yields (30).

¹³See Dinopoulos and Unel (2009) for details on the properties of $H(\lambda, \alpha)$.

where $\tilde{\lambda}$ is given by (24). It follows from (32) that market profitability can be measured by the average markup $1 - \tilde{\lambda}^{-1}$, which is *time-invariant* and increasing in $\tilde{\lambda}$.¹⁴

In Melitz (2003) and Dinopoulos and Unel (2009), the absence of economic growth and consumer savings implies that, in the steady-state equilibrium, aggregate profits finance R&D investment, and therefore, per-capita expenditure equals the wage of labor, i.e., $E = w = 1$. In the present model, consumers save, the market interest rate depends on endogenous growth ($r = \rho - \gamma$), and therefore per-capita expenditure can be written as¹⁵

$$E = \frac{(\rho - n)\bar{f}}{(1 + \theta)(1 - \tilde{\lambda}^{-1})}. \quad (33)$$

Per-capita consumption expenditure E increases in the effective discount rate $\rho - n$ and the expected innovation costs \bar{f} ; and decreases in the degree of international knowledge spillovers θ and market profitability measured by the average markup $1 - \tilde{\lambda}^{-1}$.

3 Fully-Endogenous Global Growth

Substituting (33) into the full employment condition (25) delivers the following expression for long-run growth of variety accumulation

$$\gamma = \frac{1 + \theta}{\bar{f}} - \left[\frac{1}{1 - \tilde{\lambda}^{-1}} - 1 \right] (\rho - n). \quad (34)$$

This equation identifies four distinct forces that determine the level of long-run growth. First, growth increases in the intensity of international knowledge spillovers θ , because the latter raises the productivity of R&D researchers. Second, growth decreases in the effective discount rate $\rho - n > 0$, because a higher discount rate decreases the benefits of variety

¹⁴To see that the average markup is $1 - \tilde{\lambda}^{-1}$, notice that each incumbent firm's domestic markup (measured by the price marginal-cost margin) is given by $(p - 1)/p = 1 - \lambda^{-1}$. Integrating over all such firms yields that the average domestic markup (denoted by pcm_d) is equal to $1 - \tilde{\lambda}_d^{-1}$. Since an exporter incurs a marginal cost τ , it earns a price marginal-cost margin $(p - \tau)/p = 1 - \tau\lambda^{-1}$. Hence, the average export markup (denoted by pcm_x) is equal to $pcm_x = 1 - \tau\tilde{\lambda}_x^{-1}$. It follows that the average markup is $[M_d pcm_d + M_x pcm_x]/[M_d + M_x] = 1 - \tilde{\lambda}^{-1}$.

¹⁵The free-entry innovation condition (17) implies that aggregate discounted profits must equal to aggregate instantaneous innovation costs: $\Pi/(\rho - n) = M_d \bar{f} \phi$. Substituting (32) and (10) delivers equation (33).

creation by lowering the level of expected discounted profits. Third, an increase in the (endogenously-determined) expected instantaneous fixed costs of variety creation \bar{f} reduces growth by increasing the costs of variety creation. Finally, for a given level of \bar{f} , a rise in the average markup (captured by an increase in $\tilde{\lambda}$) raises long-run growth because an increase in $\tilde{\lambda}$ increases the per-capita resources devoted to R&D by lowering the per-capita labor allocated in manufacturing $E/\tilde{\lambda}$.

We can now analyze the long-run growth effects of trade liberalization. To this end, we first start with addressing the general equilibrium effects of a move from autarky to (restricted) trade. As the level of trade costs approaches infinity (i.e., $\tau \rightarrow \infty$), the domestic cutoff quality level approaches its closed-economy value λ_d^A and its value is determined by $f_d H(\lambda_d^A, 1) = f_e$. A comparison of $f_d H(\lambda_d^A, 1) = f_e$ with (30) reveals that the autarkic cutoff quality level λ_d^A is strictly less than the open-economy domestic cutoff quality level λ_d .

Exposure to trade provides new profit opportunities to firms producing higher quality products, which in turn raises the demand for labor at any wage level. The increased demand for labor induces a reallocation of resources from lower-quality towards high-quality products. To see this note that any given aggregate supply of labor can sustain more higher-quality products, because for any level of expenditure, a firm with a higher quality product charges a higher price, produces less output, and employs less labor than a firm with a lower quality product (see equation (5)). This reallocation of labor also intensifies the product market competition by reducing the demand for each product as the aggregate expenditure EL is spread among more varieties. Thus, the net discounted value of profits of the firm with product quality level λ_d^A become less than the domestic market adaptation cost, and induce an increase in the domestic cutoff quality level to restore the cutoff condition given by (11).

Having addressed the effects of exposure to trade on the domestic quality cutoff level, we can now investigate its impact on the average markup $1 - \tilde{\lambda}^{-1}$ and the growth rate γ . Exposure to trade increases the domestic cutoff level, which in turn has a positive effect

on the average markup. However, an inspection of equation (24) indicates that when the trade costs τ or f_x is sufficiently high, $\tilde{\lambda}$ can be lower than $\tilde{\lambda}^A$. Consequently, moving from autarky to trade has an ambiguous effect on the markup.¹⁶

Setting $\zeta_x = 0$, equations (18) and (24) deliver $\bar{f}^A = f_e + f_p/[1 - G(\lambda_d^A)]$ and $\tilde{\lambda} = \tilde{\lambda}_d^A$. As a result, the closed-economy growth rate is given by

$$\gamma^A = \frac{1}{\bar{f}^A} - \left[\frac{1}{1 - (\tilde{\lambda}^A)^{-1}} - 1 \right] (\rho - n). \quad (35)$$

Subtracting this equation from (34) yields

$$\gamma - \gamma^A = \left[\frac{1 + \theta}{\tilde{f}} - \frac{1}{\bar{f}^A} \right] - \left[\frac{1}{1 - \tilde{\lambda}^{-1}} - \frac{1}{1 - (\tilde{\lambda}_d^A)^{-1}} \right] (\rho - n). \quad (36)$$

Because the expected instantaneous fixed costs of variety creation under trade exceed the corresponding costs under autarky (that is, $\bar{f} > \bar{f}^A$), the sign of the first term on the right-hand-side of (36) is ambiguous. In addition, since exposure to trade has an ambiguous effect on the average markup, the sign of the second term on the right-hand-side is also ambiguous. Thus, a move from autarky to trade has an ambiguous impact on long-run growth.

The model is also well-suited to address the growth effects of trade liberalization measured by further exposure to trade (i.e., a reduction in τ or f_x). Indeed, equations (28) and (30) have identical forms with equations (9) and (17) in Dinopoulos and Unel (2009) who demonstrate that a reduction in trade costs increases λ_d , decreases λ_x , and consequently, increases ζ_x . Thus, further exposure to trade has an ambiguous effect on $\tilde{\lambda}$ (see (24)),¹⁷ which in turn ensures that exposure to trade has an ambiguous effect on the average markup $1 - \tilde{\lambda}^{-1}$.

¹⁶Results in Harrison (1994) support this finding. Using plant-level data from Cote d'Ivoire, she finds that markups, measured as profits over sales, increased in five out of nine sectors and fell in the rest following the 1985 trade liberalization reform (see Table 5 in Harrison, 1994).

¹⁷Using (22) together with $d\lambda_d/d\tau < 0$, $d\lambda_d/df_x < 0$, $d\lambda_x/d\tau > 0$, and $d\lambda_x/df_x > 0$ implies that $d\tilde{\lambda}_d^{-1}/d\tau > 0$, $d\tilde{\lambda}_d^{-1}/df_x > 0$, $d[\zeta_x \tilde{\lambda}_x^{-1}]/d\tau < 0$ and $d[\zeta_x \tilde{\lambda}_x^{-1}]/df_x < 0$. Thus, from equation (24) it follows that the signs of $d\tilde{\lambda}/d\tau$ and $d\tilde{\lambda}/df_x$ are ambiguous.

An inspection of equation (18) indicates that a reduction in variable trade costs τ increases the expected instantaneous costs \bar{f} . However, a reduction in foreign market-entry costs f_x has an ambiguous effect on the expected instantaneous costs \bar{f} , because while $f_e/[1 - G(\lambda_d)]$ and ζ_x increase, f_x decreases.

Equation (34) reveals that long-run growth increases in the intensity of trade-related knowledge spillovers θ .¹⁸ Combined with Proposition 1 and our above discussion, we can now state the effects of trade liberalization on long-run growth.

Proposition 2. *A move from autarky to (restricted) trade or further exposure to trade, captured by a reduction in variable trade costs ($\tau \downarrow$) or a reduction in foreign-market entry costs ($f_x \downarrow$) increases the domestic cutoff quality level λ_d ; and has an ambiguous on the average markup and long-run growth rate γ . Furthermore, long-run growth γ increases in the intensity of trade-related knowledge spillovers θ .*

The result that exposure to trade has an ambiguous effect on long-run growth is similar to that in Baldwin and Robert-Nicoud (2008) and Unel (2008). However, there are two important differences between our result and theirs. First, the long-run growth in their model becomes infinite in the presence of positive population growth, that is, their model is subject to the scale effects critique of Jones (1995a). Second, in their model the ambiguous relationship between trade liberalization and growth stems solely from the effect of technology diffusion through trade. If trade is not a conduit of technology transfer (i.e., $\theta = 0$), exposure to trade in their models unambiguously reduces long-run growth. In contrast, our model highlights the feature that, even in the absence of technology diffusion through trade, exposure to trade has an ambiguous effect on long run growth. The latter stems from the presence of limit pricing and variable markups. In contrast, models based on Dixit-Stiglitz (1977) preferences deliver constant markups.

The ambiguous growth effect of trade liberalization is absent from the semi-endogenous growth model with heterogeneous firms proposed by Gustafsson and Segerstrom (2007),

¹⁸The equilibrium values of λ_d , λ_x , \bar{f} , and \tilde{f} are independent of θ .

where long-run growth depends positively only on the rate of population growth and a parameter measuring the intensity of intertemporal knowledge spillovers. Since the equilibrium values of \bar{f} and $\tilde{\lambda}$ are independent of the rate of population growth n , long-run growth in our model depends positively on the rate of population growth (see (34)): an increase in the rate of population growth lowers the effective discount rate $\rho - n$, reduces consumption expenditure and shifts resources from manufacturing towards R&D investment. This resource shift raises the long-run rate of economic growth.

Finally, our results complement those in Haruyama and Zhao (2008) who develop first generation and semi-endogenous quality ladders growth models with heterogeneous firms, where endogenous growth is based on quality improvements (as opposed to variety expansion) and heterogeneity is based on productivity differences among firms. In their model, firms use limit-price strategies that generate endogenous markups as in our model. They find that trade liberalization increases average markups and the share of labor used in R&D. According to their R&D specification that delivers endogenous growth (with scale effects), trade liberalization unambiguously raises long-run growth. However, when the model is modified to remove the scale effect, they find that the rate of technical progress is independent of trade liberalization –the long run growth rate in their semi endogenous growth model is driven by the rate of population growth as in Gustafsson and Segerstrom (2007). In contrast, the present paper present a fully endogenous growth model in which the effects of trade liberalization on the average markup and the instantaneous fixed R&D cost are ambiguous. This in turn delivers an ambiguous effect on the long-run growth rate of technical progress.

4 Welfare Analysis

We now analyze the welfare properties of the model. We would like to address two important questions. First, what is the dynamic effect of trade liberalization on consumer welfare? Second, is the market resource allocation socially optimal? Following the relevant literature,

we measure welfare by the instantaneous utility function of the representative consumer. Since $p(\lambda) = \lambda$, (3) implies that the corresponding per-capita demand is given by $\lambda q(\lambda)/L = E/M_c$. Inserting this expression into the utility function defined by (2) and performing the integration yields

$$\ln u = M_c \ln(E). \quad (37)$$

Observe that utility of a typical consumer depends positively on the mass of varieties consumed M_c and on the per-capita expenditure E_t .¹⁹ Since in the steady-state equilibrium E_t is time-invariant (see equation (34)), equation (37) implies that the (log) instantaneous utility grows at rate γ . Combined with Propositions 2 and 3, we establish the following result.

Proposition 3. *A reduction in variable trade costs ($\tau \downarrow$) or foreign entry cost ($f_x \downarrow$) has an ambiguous effect on long-run welfare growth, whereas an increase in the intensity of trade-related knowledge spillovers raises long-run welfare growth.*

We are now in a position to demonstrate that the market cutoff quality levels and growth rate are socially suboptimal. For the sake of brevity, we shall only consider the closed economy case. In this case, number of products consumed will be the same as the number of products produced, i.e., $M_c = M_d = M$. The planner maximizes the following intertemporal utility function

$$U = \int_0^\infty e^{-(\rho-n)t} M(t) \ln[E(t)] dt, \quad (38)$$

subject to the resource constraint which reflects the full-employment of labor condition. Before going further, it is important to notice that we must have $\ln[E(t)] > 0$ (or $E > 1$); otherwise, consuming all available products will not be an optimal decision.

¹⁹We can now derive a sufficient condition which guarantees that the consumer consumes all available varieties. In principle, each consumer chooses $q(\omega)$ and M to maximize (2) subject to the budget constraint. The first order condition with respect to M yields $\ln[M_c \lambda(M) q(M)/L] \geq 0$, which holds with equality if $M < M_c$. Using $p(M) = \lambda(M)$ and (3) in this condition yields $\ln E \geq 0$. Thus to ensure that $M = M_c$, we must have that $E > 1$. Since $\theta \in [0, 1]$, $\zeta_x \in [0, 1]$, and $\lambda_p \in (1, k)$, using (34) together with $E > 1$ implies $f_x > 2/(\rho - n)$.

The full employment condition (25) implies that total labor used in the R&D process is given by $L_e = L - \tilde{\lambda}_d^{-1}EL$, where $\tilde{\lambda} = \tilde{\lambda}_d$. Inserting this into (19) delivers

$$\dot{M} = (1 - \tilde{\lambda}_d^{-1}E)M/\bar{f}, \quad (39)$$

where $\bar{f} \equiv f_e/[1 - G(\lambda_d)] + f_d$.

In the steady-state equilibrium, E and λ_p are time-invariant and the socially optimum values E^S and λ_d^S satisfy the following system of equations (see Appendix for details):

$$f_d H(\lambda_d, E) = f_e, \quad (40)$$

$$1 - \tilde{\lambda}_d^{-1}E(1 - \ln E) = (\rho - n)\bar{f}, \quad (41)$$

where $H(\lambda_d, E)$ is defined by (31) and the requirement that the denominator $1 - \lambda_d^{-1}E$ in $H(\lambda_d, E)$ must be positive implies that $\lambda_d > E$. Furthermore, $H(\lambda_d, E)$ decreases in λ_d and increases in E .²⁰

Equations (40) and (41) define two curves in (λ_d, E) space (not shown for brevity of exposition). The slope of the curve defined by (40) is positive, while the slope of (41) is indeterminate.²¹ Consequently, these curves may intersect each other more than once, and hence, might give rise to multiple socially optimum cutoff quality levels.

How does the socially optimum solution compare to the market equilibrium? To facilitate the comparisons of the efficient and market equilibria, rewrite the conditions that determine the market values of E^A and λ_d^A :

$$f_d H(\lambda_d, 1) = f_e, \quad (42)$$

$$(1 - \tilde{\lambda}_d^{-1})E = (\rho - n)\bar{f}, \quad (43)$$

where (43) is obtained from the closed-equilibrium versions of (32).²² Observe that (43) and (41) will be identical, if E in (43) is equal to one. In this case, the efficient and market

²⁰ $\partial H/\partial E = [1 - G(\lambda_d)](\lambda_d^{-1} - \tilde{\lambda}_d^{-1})/(1 - \lambda_d^{-1}E)^2 > 0$.

²¹Totally differentiating (40) yields $dE/d\lambda_d = \{1 + [1 - G(\lambda_d)]f_d/f_e\}(E/\lambda_d)^2 > 0$. On the other hand, totally differentiating (41) yields $dE/d\lambda_d = \mu(\lambda_d)\tilde{\lambda}_d\{(\rho - n)f_e/[1 - G(\lambda_d)] - (\lambda_d^{-1} - \tilde{\lambda}_d^{-1})E(1 - \ln E)\}/\ln E$. Notice that for $\ln E \geq 1$, the slope is positive.

²²From the free-entry innovation condition (17), we have $\Pi/(\rho - n) = M_d\bar{f}\phi$. Substituting (32) and setting $M_c = M_d$ and $\theta = 0$ yields (43).

cutoff levels will be the same, i.e., the market cutoff level is socially optimal. However, as discussed above, in the optimal solution E^S must be greater than 1. Furthermore, since $H(\lambda_d, E)$ increases in E , it follows that $\lambda_d^S > \lambda_d^A$, i.e., the efficient cutoff level is greater than the market cutoff level. Because λ_d^S differs from λ_d^A , E^S generally differs from E^A .

Once the efficient per-capita expenditure and production cutoff level are determined, one can obtain the socially optimum growth rate by substituting the efficient solution in (39):

$$\gamma^S = \frac{1}{\bar{f}^S} - \frac{(\tilde{\lambda}_d^S)^{-1} E^S}{\bar{f}^S}, \quad (44)$$

where $\bar{f}^S = f_e/[1 - G(\lambda_d^S)] + f_d$. A comparison of (44) with (35) reveals that generally the socially optimal growth rate differs from the market growth rate.

Proposition 4. *The laissez-faire cutoff quality level λ_p^A , per-capita expenditure E , and growth rate γ^A are socially sub-optimal.*

5 Conclusion

This paper developed a fully-endogenous growth model with firm-specific quality heterogeneity and an endogenous distribution of markups and profit margins. The model was used to analyze the determinants of long-run global growth and welfare. In doing so, it complements recent studies of growth with firm-specific productivity heterogeneity. The effects of trade liberalization on long-run growth work through the incentives to innovate. An increase in the rate of international knowledge spillovers unambiguously increased the rate of long-run growth by increasing the productivity of R&D researchers. However, a reduction in trade costs, or a reduction in foreign-market entry costs, generate pro-growth and anti-growth forces. The former stem from an increase in the average profit margin which raises the benefits of R&D, whereas the latter arise from the possibility that R&D can become more risky and be associated with higher expected instantaneous fixed costs. This source of ambiguity which can be traced to the forces that deliver firm heterogeneity

(trade and foreign-market entry costs) is a novel addition to the literature of growth with heterogeneous firms.

We conclude by identifying a few areas for further research. First, the theory fully-endogenous growth has enjoyed empirical support and the same holds for the theory of trade with heterogeneous firms. Therefore, by unifying the two areas, this paper is the first to offer a novel theoretical framework that can generate a number of testable hypotheses concerning the nexus among growth, trade, and firm heterogeneity. In addition, empirical work could, at least in principle, resolve the inherent ambiguity associated with the effects of trade liberalization on growth. Second, although the assumption of two structurally identical countries simplifies the algebra considerably, it is clearly not realistic. The introduction of asymmetric countries would allow the development of North-South models of growth with heterogeneous firms and the study of several questions related to economic development. Finally, one could introduce another factor of production and analyze the effects of trade liberalization on growth and income distribution in the presence of heterogeneous firms.

Appendix: Analysis of The Planner's Problem

The corresponding Hamiltonian is given by

$$\mathcal{H}(E, \lambda_d, M) = e^{-(\rho-n)t} M \ln E + \psi \left[\frac{1 - \tilde{\lambda}_d^{-1} E}{\tilde{f}} \right] M,$$

where ψ is a time-varying Lagrange multiplier. In this problem, E and λ_d are control variables, while M is a state variable. The necessary conditions are given by

$$\frac{\partial \mathcal{H}}{\partial E} = 0 \implies e^{-(\rho-n)t} \tilde{f} = \psi \tilde{\lambda}_d^{-1} E, \quad (45)$$

$$\frac{\partial \mathcal{H}}{\partial \lambda_d} = 0 \implies [1 - G(\lambda_d)] \left[\frac{1 - \tilde{\lambda}_d^{-1} E}{1 - \lambda_d^{-1} E} - 1 \right] = \frac{f_e}{f_d}, \quad (46)$$

$$\frac{\partial \mathcal{H}}{\partial M} + \dot{\psi} = 0 \implies \frac{e^{-(\rho-n)t} \ln E}{\psi} + \frac{1 - \tilde{\lambda}_d^{-1} E}{\tilde{f}} = -\frac{\dot{\psi}}{\psi}, \quad (47)$$

$$\lim_{t \rightarrow \infty} \psi(t) M(t) = 0. \quad (48)$$

The last condition represents the transversality condition.

Again, we shall only consider the steady-state equilibrium. Notice that $1 - \tilde{\lambda}_p^{-1}E$ represent the fraction of labor used in the R&D process and in the steady-state equilibrium, this must be constant. Moreover, since \dot{M}/M is constant in the steady-state, the constraint implies that \bar{f} also must be constant. This further implies that λ_p must be constant. Finally, combining this with the constancy of $1 - \tilde{\lambda}_p^{-1}E$ ensures that E is constant.

Notice that (46) is the same as (40). Since λ_d and E are time-invariant, (45) implies that ψ grows at rate $n - \rho$. Moreover, solving (45) for ψ and substituting it into (47) implies (41) in the main text.

References

- Aghion, Philippe and Peter Howitt, “A Model of Growth Through Creative Destruction,” *Econometrica*, March 1992, *60*, 323–51.
- Aw, Bee Yan, Sukkyun Chung, and Mark J. Roberts, “Productivity and Turnover in the Export Market: Micro Evidence from the Republic of Korea and Taiwan (China),” *The World Bank Economic Review*, 2000, *14*, 65–90.
- Baldwin, Richard E. and Frederic Robert-Nicoud, “Trade and Growth with Heterogeneous Firms,” *Journal of International Economics*, 2008, *74*, 21–34.
- Bernard, Andrew B. and J. Bradford Jensen, “Exceptional Exporter Performance: Cause, Effect, or Both,” *Journal of International Economics*, 1999, *47*, 1–25.
- ____ and _____, “Exporting and Productivity in the USA,” *Oxford Review of Economic Policy*, 2004, *20*, 343–57.
- Clerides, Sofronis K., Saul Lach, and James R. Tybout, “Is Learning by Exporting Important? Micro-Dynamic Evidence from Columbia, Mexico, and Morocco,” *Quarterly Journal of Economics*, 1998, *113*, 903–47.

- Coe, David T. and Elhanan Helpman, “International R&D Spillovers,” *European Economic Review*, May 1995, *39*, 859–87.
- Dinopoulos, Elias and Bulent Unel, “A Simple Model of Quality Heterogeneity and International Trade,” 2009. University of Florida, Working Paper.
- and Paul Segerstrom, “A Schumpeterian Model of Protection and Relative Wages,” *American Economic Review*, 1999, *89*, 450–72.
- and Peter Thompson, “Schumpeterian Growth Without Scale Effect,” *Journal of Economic Growth*, 1998, *3*, 313–35.
- Dixit, Avinash K. and Joseph E. Stiglitz, “Monopolistic Competition and Optimum Product Diversity,” *American Economic Review*, 1977, *67*, 297–308.
- Grossman, Gene M. and Elhanan Helpman, “Innovation and Growth in the Global Economy,” 1991.
- Gustafsson, Peter and Paul Segerstrom, “Trade Liberalization and Productivity Growth,” 2007. Mimeo, Stockholm School of Economics.
- Ha, Joonkyung and Peter Howitt, “Accounting for Trends in Productivity and R&D: A Schumpeterian Critique of Semi-Endogenous Growth Theory,” *Journal of Money, Credit, and Banking*, June 2007, *33*, 733–74.
- Hallak, Juan C., “Product Quality and the Direction of Trade,” *Journal of International Economics*, 2006, *68*, 238–65.
- Harrison, Ann, “Productivity, Imperfect Competition, and Trade Reform: Theory and Evidence,” *Journal of International Economics*, 1994, *36*, 53–73.
- Haruyama, Tetsugen and Laixun Zhao, “Trade and Firm Heterogeneity in A Quality-Ladders Model of Growth,” 2008. Mimeo, Kobe University.

- Howitt, Peter, “Steady Endogenous Growth with Population and R&D Inputs Growing,” *Journal of Political Economy*, 1999, *107*, 715–30.
- Hummels, David and Alexandre Skiba, “Shipping the Good Apples Out? An Empirical Confirmation of the Alchian-Allen Conjecture,” *Journal of Political Economy*, 2004, *112*, 1384–1402.
- and Peter J. Klenow, “The Variety and Quality of a Nation’s Exports,” *American Economic Review*, 2005, *95*, 704–23.
- Jones, Charles I., “Times Series Tests of Endogenous Models,” *Quarterly Journal of Economics*, 1995a, *110*, 495–525.
- , “R&D Based Models of Economic Growth,” *Journal of Political Economy*, 1995b, *103*, 759–84.
- Keller, Wolfgang, “International Technology Diffusion,” *Journal of Economic Literature*, 2004, *42*, 755–82.
- Kortum, Samuel, “Research, Patenting, and Technological Change,” *Econometrica*, 1997, *65*, 1389–1419.
- Lucas, Robert E., “On the mechanics of economic development,” *Journal of Monetary Economics*, July 1988, *22*, 3–42.
- Melitz, Marc J., “The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity,” *Econometrica*, November 2003, *71*, 1695–1725.
- Rivera-Batiz, Luis A. and Paul M. Romer, “Economic Integration and Endogenous Growth,” *Quarterly Journal of Economics*, May 1991, *106*, 531–55.
- Roberts, Mark J. and James R. Tybout, “The Decision to Export in Columbia: An Empirical Model of Entry with Sunk Costs,” *American Economic Review*, September 1997, *87*, 545–64.

- Romer, Paul M., “Endogenous Technical Change,” *Journal of Political Economy*, October 1990, *98*, 71–102.
- , “New Goods, Old Theory, and the Welfare Costs of Trade Restrictions,” *Journal of Development Economics*, 1994, *43*, 5–38.
- Schott, Peter K., “Across-Product Versus Within Product Specialization in International Trade,” *Quarterly Journal of Economics*, 2004, *119*, 647–78.
- Segerstrom, Paul, “Endogenous Growth Without Scale Effects,” *American Economic Review*, 1998, *88*, 1290–1310.
- , T. C. A. Anant, and Elias Dinopoulos, “A Schumpeterian Model of the Product Life Cycle,” *American Economic Review*, 1990, *80*, 1077–92.
- Unel, Bulent, “Technology Diffusion Through Trade with Heterogeneous Firms,” 2008. *Review of International Economics*, Forthcoming.
- Young, Alwyn, “Growth Without Scale Effects,” *Journal of Political Economy*, February 1998, *106*, 41–63.