



DEPARTMENT OF ECONOMICS WORKING PAPER SERIES

Method to Find the VARs Easily

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Working Paper 2006-11
http://www.bus.lsu.edu/economics/papers/pap06_11.pdf

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Method to find the VARs easily.

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May 2006 (preliminary)

Abstract

The paper shows an easy method to get the impulse responses of VARs of a stochastic recursive dynamic macro model by defining the transition matrix and the stationary distribution function of a model using the model, i.e. economic theory, itself.

JEL Classification: C15, C32, C61, E17.

Keywords: Markov Equilibria, transition matrix, VARs, economic model.

*Department of Economics, Louisiana State University, abirk@lsu.edu. I thank Bart Taub for support and many helpful comments at the initial stage of my current research area.

1 Introduction

In the paper, I apply the method of dynamic stochastic programming developed in recent years by Lucas and others¹ to the labor market and show that the stochastic transition matrix of an economic model can be defined in such a way that it is a squared matrix and invertible. This might be helpful for empirical researchers, when they want to estimate the impulse response of a vector autoregression (VAR).² In our dynamic stochastic macroeconomic models, the properties for a stochastic transition matrix to be a squared matrix and invertible are usually fulfilled, which I will show by discussing two examples.

The examples are set up in a search economic environment in which unemployed workers are looking for jobs. Using the method of stochastic dynamic programming, reservation decisions about accepting job offers will be derived endogenously. With the simulated optimal policy functions, which result from simulating the worker's intertemporal maximization problem and which characterize the thresholds of accepting or rejecting a wage offer, an *endogenous squared transition matrix* will be defined.

I will discuss next that the squared transition matrix is *invertible* and, due to this property together with the assumption of a finite state space, an invariant distribution over states will be defined as well as simulated and characterizes the stationary equilibrium of the economy. That is, starting from the worker's different transition states, which are summarized by the squared Markov matrix, the economy converges to the invariant distribution, reflecting the various population fractions for different states. The iterations of the transition matrices reflect the model's impulse-responses over time. In my chosen search economic environment, the invariant distribution describes the fractions of employed or unemployed workers in specific states. Taking the discrete state space into account, different stationary equilibria will be determined.

Therefore, the paper aims to show how the squared transition matrix together with the identity matrix can describe the impulse-responses over time of an economic model and the calibrated impulse-responses can then be compared to those generated by empirical researchers. That is, developing a method of how to define and get a squared and invertible transition matrix from economic theory might be useful in order to compare the theoretical impulse-responses with those that result when estimating the impulse responses of VARs.

The next section provides a first example of an economy in which a worker is searching for a job, and it defines the transition probability matrix as well as the stationary equilibrium for the economy. In Section 3, the simulation procedure is described and subsections state the calibration results for this labor market example. In Section 4, an open economy will be set up, in which workers can be unemployed or employed and searching for jobs in two countries. The subsections define the transition matrix as well as the stationary labor market for the source country and discuss the calibration results. Section 5 concludes.

2 An economy in a first example

In the first example, the economy is populated with a finite number of ex ante identical but ex post heterogeneous workers who can be employed or unemployed. There is no aggregate uncertainty and no variation of an aggregate state variable over time, but much uncertainty at the individual level.³ The individuals face a version of an infinite horizon search problem. Their option is to manage their state of employment by accepting or rejecting a job offer, when facing wage and employment shocks. The model uses the previous employment state as a vehicle of insurance.

¹See for example Stokey, Lucas (1989).

²For a more detailed discussion of that relationship see for example Fernandez-Villaverde, Rubio-Ramirez and Sargent (2005).

³For a similar approach see Ljungqvist, Sargent (1998, 2005).

2.1 The employment decision of a representative worker

Consider a representative worker. The worker sees her new skill level at the beginning of a period, searches for a job in this period and is employed or unemployed in the next period. That is, at the beginning of each period t , the worker is unemployed and chooses search effort $e_t \geq 0$ to look for a job. Search induces disutility $c(e)$, which is increasing in e and may lead to a wage offer. With probability $\pi(e_t)$, $\pi_{e_t} > 0$, the worker draws a wage offer in $t + 1$. The wage offers are drawn from the time-invariant wage distribution $F(w) = \Pr(w_{t+1} \leq w)$. The set of possible wage values is denoted W , $W = \{w_1, \dots, w_l\}$. With the wage offer at hand, the worker can accept or reject the job offer. Accepting a offer implies the worker will be employed in period $t + 1$. Each wage follows a Markov process with stationary transition probability $G(w'|w) = \Pr(w_{t+1} \leq w' | w_t = w) > 0$, $\forall w, w' \in W$, independent of all other wage offers. Rejecting the wage offer means the worker is unemployed in period $t + 1$. With probability $(1 - \pi(e_t))$, the worker will not receive a wage offer at all.

The worker's skills can stochastically depreciate or accumulate, depending on whether she is employed, unemployed or laid off. The skills of an unemployed worker depreciate with probability $\mu_u(h, h')$ from h to h' , where $h, h' \in H$ is the skill level. For a laid-off worker, her skills depreciate with probability $\mu_l(h, h')$ in the initial laid-off period and with the probability $\mu_u(h, h')$ afterwards. However, if the worker is employed, she accumulates skills with probability $\mu_e(h, h')$ until she reaches the highest skill level or becomes unemployed. The worker's skill level stays the same when moving to the other country.

Once the worker has quit the job or is unemployed, she can be entitled for social assistance. On the one hand, she will be eligible for unemployment benefits $b(I)$, if the government's suitable earning criterion $I_e(I)$ is higher than the worker's previous earned wage income $I (= wh)$. On the other hand, as long as the government's suitable earnings are lower than last earnings, no benefits are paid at all. Net benefits are $(1 - \tau)b(I)$ with τ as the tax rate.

When the worker was previously employed, she can be laid off with probability $\lambda \in [0, 1]$ and is unemployed in period t . Let K be the layoff costs when being laid off or having quit the job. Laid off workers can qualify for unemployment compensation too, if the foregone earnings fall short of the government's suitable earnings criterion. That is, as long as $wh < I_u(I)$, the worker is eligible for benefits.

At each point in time, the worker's decision problem is described by an individual state vector $s \in S$ with $s = (h, I)$ and S is the individual state space with $S = H \times W$ for $H = \{h_1, \dots, h_m\}$ and $W = \{w_1, \dots, w_l\}$. On the basis of this state vector, the worker makes the employment decision.

After the characterization of the worker's environment, her functional equations will be defined. Let $V(h, w, I)$ be the value of the optimization problem for the worker with skill level h and wage w who was employed and earned income I in the previous period and who decides upon a specific wage offer to accept the job. $V_b(h, I_k)$ is the value of the optimization problem for an unemployed worker with skill level h and last earnings I who is entitled for unemployment compensation. $V_0(h)$ is the value for an unemployed worker who is not entitled to unemployment compensation. The Bellman equations for this sequential search problem are:

$$V(h, w, I) = \max_{\text{accept, reject}} \{ \Omega(h, w), D(h, w, I)V_b(h, I) + (1 - D(h, w, I))V_0(h) - K \}, \quad (1)$$

$$\begin{aligned}
V_b(h, I) = & \max_e \left\{ -c(e) + (1 - \tau)b(I) + \beta \sum_{h'} \mu_u(h, h') \right. \\
& [(1 - \pi(e))V_b(h', I) + \pi(e) * \\
& \left. \left(\int_{w < I_u(I)/h'} \max_{\text{accept, reject}} \{ \Omega(h', w), V_b(h', I) \} dF(w) \right. \right. \\
& \left. \left. + \int_{w \geq I_u(I)/h'} \max_{\text{accept, reject}} \{ \Omega(h', w), V_0(h') \} dF(w) \right) \right\}, \tag{2}
\end{aligned}$$

$$\begin{aligned}
V_0(h) = & \max_e \left\{ -c(e) + \beta \sum_{h'} \mu_u(h, h') [(1 - \pi(e)) \right. \\
& \left. V_0(h') + \pi(e) \int \max_{\text{accept, reject}} \{ \Omega(h', w), V_0(h') \} dF(w) \right\}, \tag{3}
\end{aligned}$$

where

$$\begin{aligned}
\Omega(h, w) \equiv & (1 - \tau)wh + \beta \left[\lambda \sum_{h'} \mu_l(h, h') V_b(h', wh) - \lambda K + \right. \\
& \left. (1 - \lambda) \sum_{h'} \mu_e(h, h') \int V(w', h', wh) dG(w'|wh) \right], \\
D(h, w, I) = & \begin{cases} 1, & \text{if } w < I_e(I)/h, \\ 0, & \text{if } w \geq I_e(I)/h, \end{cases}
\end{aligned}$$

The intertemporal optimization problem (1) – (3) can be solved numerically. This gives functions, first, for the optimal search intensities $e_b^*(h, I)$ and the reservation wages $\bar{w}_b(h, I)$ for an unemployed worker who is eligible for unemployment compensation, second, for the optimal search intensities $e_0^*(h)$ and the reservation wages $\bar{w}_0(h)$ for an unemployed worker who is not entitled for benefits and, third, the reservation wages $\bar{w}(h, w, I)$ for an employed worker. The functions $\bar{w}_0(h) : S \rightarrow \mathbb{R}$, $\bar{w}_b(h, I) : S \rightarrow \mathbb{R}$ and $\bar{w}(h, w, I) : S \rightarrow \mathbb{R}$ are optimal decision rules provided they are measurable, feasible and satisfy the functional equations as optimal value functions.⁴

2.2 The stationary equilibrium for the labor market

With the optimal decision rules at hand, I define the stationary equilibrium for the labor market, show examples of transition probability matrices and derive invariant distribution functions for various states. The state space is S .

Since workers will be heterogeneous in their individual state vectors, a way of describing the heterogeneity at a point in time is needed. A probability measure defined on subsets of the individual state space is a natural way of describing this heterogeneity. Let S be the finite set with $\{s_1, \dots, s_r\}$ and ψ be a probability measure on (S, \mathcal{S}) , where $S = H \times W$ and \mathcal{S} is the Borel σ -algebra. Thus, for $S \in \mathcal{S}$, $\psi(S)$ indicates the mass of workers, whose individual state vectors lie in S . Notice also that $F(0) = 0$ and $F(w) = 1$.

The aggregate state of the economy is given by ψ . As ψ changes over time, wages might change too. However, wages and the wage distributions are assumed to be constant here. For many questions, the dynamics caused by changing distributions of individual state vectors are of interest. For the question at hand, I define a more specialized notion of an equilibrium, in which the probability measure ψ remains unchanged over time.

⁴See Theorem 9.2 in Stokey and Lucas (1989).

An important technical reason for concentrating on stationary equilibria is that methods for characterizing equilibria in general, do not exist currently. The paper adopts, therefore, the stationary recursive equilibrium structure described in Stokey and Lucas (1989, p.320f).⁵ To define what it means for a probability measure ψ to be stationary over time, a transition function $\mathcal{P}, \mathcal{P} : S \times \mathcal{S} \rightarrow [0, 1]$, is needed. Intuitively, $\mathcal{P}(s, S)$ is the probability that a worker with state s will have an individual state vector lying in S in the next period. The next paragraph shows how to construct the transition probabilities $\psi(\bar{w}'(s)|\bar{w}(s))$ and a Markov transition probability matrix from the decision rules $\bar{w}_0(s), \bar{w}_b(s), \bar{w}(s)$. Equipped with a well-defined transition function \mathcal{P} , a probability measure ψ defined on (S, \mathcal{S}) is stationary.

Definition 1 *A stationary recursive equilibrium for the labor market of this economy are reservation wage functions $(\bar{w}_0(s), \bar{w}_b(s), \bar{w}(s))$, and distribution functions $\psi(s)$ such that*

- (1) $\bar{w}_0, \bar{w}_b, \bar{w}$ solve the worker's optimization problem (1)–(3)
- (2) an n -dimensional vector $s \in R^n$ records the possible values of the state of the system;
- (3) an $(n \times 1)$ vector ψ_0 records the probabilities of being in each state i at time 0 with
 - (a) $\psi_{0i}(S) = \Pr[w_0(s) = \bar{w}_i(s)]$,
 - (b) $\psi_0(S) = \sum_{i=1}^n \psi_{0i}(s) = 1$;
- (4) an $n \times n$ transition matrix $\mathcal{P}(s, S)$ records the probabilities of moving from one value of the state to another in one period;
- (5) $\psi(s)$ is time-invariant.

The first condition says workers optimize. The third condition defines the worker's unconditional initial probabilities and the initial probability distribution over the initial state, the fourth defines the transition probability matrix, and the fifth defines a stationary equilibrium by stating the distribution of workers over states to be constant.

I compute equilibria for the country's labor market and restrict the analysis to situations, in which a worker is eligible for unemployment compensation only. The reservation wage $\bar{w}_b(s)$ remains. With these functions at hand, the partial equilibrium analysis can continue.

The following characterizes a worker's initial probability distribution over states, i.e. situations in which the worker accepts a wage offer or rejects it and remains unemployed. That is, with the reservation wage of being eligible for unemployment benefits, $\bar{w}_b(s)$, a worker can be in one of two states

$$\psi_{0i}(s) = \begin{cases} F(\bar{w}_b(s)), & \bar{w}_b(s) > w, \\ (1 - F(\bar{w}_b(s))), & \bar{w}_b(s) \leq w, \end{cases} \quad (4)$$

The decision rule defines the worker's unconditional probabilities $\psi_{0i}(s)$ at time 0 for $i = 1, 2$. In the first initial state, the worker rejects a wage offer and stays unemployed, whereas in the second initial state, $i = 2$, the worker accepts a wage offer and is employed with probability $(1 - F(\bar{w}_b(s)))$. The unconditional probabilities are used to define the different states in the transition matrix, in which workers can be. I do this next.

⁵See especially Theorem 11.1 in Stokey, Lucas (1989).

The Markov transition matrix defines the conditional transition probabilities of moving from one state to another, which include the endogenous unconditional probabilities as well as the exogenous probabilities. The transition probabilities are conditional on the employment state of the current period.

Proposition 2 *Let $\mathcal{P}_{ij} = \mathcal{P}(s_i, \{s_j\})$ be the conditional probability of being currently in state i and move to state j in the next period, then the transition probabilities of the states are defined as*

$$\mathcal{P}_{ij} = \Pr[w_{t+1}(s) = \bar{w}_j(s) | w_t(s) = \bar{w}_i(s)] = \begin{pmatrix} \pi F + (1 - \pi) & \pi(1 - F) \\ \lambda + (1 - \lambda)F & (1 - \lambda)(1 - F) \end{pmatrix}. \quad (5)$$

Proof. Since $\mathcal{P}_{ij} \geq 0$ and $\sum_{j=1}^n \mathcal{P}_{ij} = 1$ for $i = 1, \dots, n$, \mathcal{P} is an $n \times n$ Markov matrix. ■

Here $n = 2$, so the transition matrix is 2×2 ,⁶ where $F \equiv F(\bar{w}_b(s))$ is the unconditional probability of being unemployed, which depends on the endogenous reservation wage of being eligible for unemployment benefits, $\pi \equiv \pi(e^*(s))$ is the endogenous probability of getting a new wage offer, which depends on the optimal search intensity, with which a worker is looking for a new job, and λ is the exogenous probability of being laid off. The rows define the two transition probabilities of staying further on unemployed or being employed in the next period, whereby the first row is conditional on being unemployed currently, and the second row is conditional on being currently employed. In the first row, the first transition probability, \mathcal{P}_{11} , is the probability of being unemployed in the next period conditional on being currently unemployed and consists of the probability of rejecting a wage offer πF plus the probability of getting no wage offer at all $(1 - \pi)$. \mathcal{P}_{12} is the conditional probability of being employed in the next period; especially it is the probability of drawing a wage offer times the probability of accepting it conditional on being currently unemployed. The second row shows the conditional probabilities of being unemployed or employed in the next period, conditional on being currently employed. \mathcal{P}_{21} is the probability of being unemployed in the next period conditional on being currently employed and defines the conditional probability as the probability of being laid off λ plus the probability of not being laid off, but rejecting the new wage offer, $(1 - \lambda)F$. Finally, \mathcal{P}_{22} is the conditional probability of becoming employed next period conditional on being employed this period and it consists especially of probability of not being laid off this period times the probability of accepting a new wage offer in the next period, $(1 - \lambda)(1 - F)$. Thus, $i = 1$ reflects the transition probabilities of being employed or unemployed in the next period conditional on being currently unemployed, and $i = 2$ states the probabilities for the next period's employment states conditional on being currently employed.

With the transition function $\mathcal{P}(s, S)$ at hand, the equilibrium is stationary and described by the invariant distribution $\psi(s)$, which is shown in the next proposition.

Proposition 3 *If $\psi(s)$ satisfies*

$$(I - \mathcal{P}')\psi = 0, \quad (6)$$

the equilibrium is stationary and characterized by the invariant distribution function $\psi(s)$.

Proof. (6) determines $\psi(s)$ as an eigenvector associated with a unit eigenvalue of \mathcal{P}' . That is, the fact that \mathcal{P} is a stochastic matrix, i.e., it has nonnegative elements and satisfies $\sum_j P_{ij} = 1$ for all i , guarantees that \mathcal{P} has at least one unit eigenvalue, and there is at least one eigenvector that satisfies equation (6). ■

The proposition says that if the transition probabilities in the stochastic matrix \mathcal{P} add up to one, which they should do in finite states and with $F(\cdot) = 1$, and if it is a squared matrix, then the transition matrix will converge to an invariant distribution ψ , reflecting the stationary steady state of the economy.

⁶In Section 4, $n = 4$ and the transition matrix is thus a 4×4 matrix.

Notice that this proposition seems to be fruitful for empirical researchers, when they try to estimate the impulse responses of VARs of stochastic recursive models. Equation (6) together with the explicitly defined transition matrix $\mathcal{P}(\cdot)$ reflect the impulse responses of the economic model and can be used by empirical researchers to compare the impulse responses of their estimated VARs with those of the economic model. Fernandez-Villaverde, Rubio-Ramirez and Sargent (2005) might have been inspired by some other papers,⁷ which show the explicit convergence of a squared matrix towards the invariant distribution in a finite state space, and they see and conclude that transition matrices having similar properties than the transition matrix \mathcal{P} can be used to estimate and compare the impulse responses of VARs. These properties are the discussed above ones, which can be summarized by, first, the transition matrix has to be a squared matrix and, second, if \mathcal{P} is a squared matrix, then the transition matrix is stable, which means that the eigenvalues are less than one in modulus and, therefore, it converges to the invariant distribution. This proposition reflects thus one of the key result of the paper.

Another result of the paper is the programming part of the model, which allows one to get the calibrated transition matrix and invariant distribution function of the model, which I will explain next.

3 Simulation Procedure

The following steps have been used to simulate the model. An important assumption is that the state space is finite.

- i. In the first step, I define all the variables, the parameters and the exogenous Markov matrices of the model.
- ii. In the second step, I program the value functions according to the theoretical model in Mathematica, where one has to add only the iteration indices to the value functions.
- iii. Then the value function iteration can be done. The functions (1) - (3) should to be programmed and then calculate for each function the norm, starting with $V(\cdot) = 0$, $V_b(\cdot) = 0$, $V_0(\cdot) = 0$ and $\Omega(\cdot) = 0$. After 300 to 500 iterations, the functions usually converge.
- iv. In the fourth step, I calculate the optimal search intensities by picking the argmax out of the search intensities from the right hand side of the final converged value functions V_b and V_0 .
- v. In the fifth step, I select the reservation wages. All these steps are well known.
- vi. In the sixth step, I define the identity matrix as well as the transition matrix. The important steps are, first, to define the transition matrix according to the model, i.e. one has to define the endogenous and exogenous probabilities in the transition matrix. In this first example, the matrix has been defined according to equation (5). Second, I programmed the transition matrix together with the identity matrix so that for each state the identity matrix picks the current state and makes all other states to zero. For each row, the various transition probabilities of the current state have to add up to one. Notice that the transition matrix has to be programmed in such a way that if the state changes, the transition probabilities have to change too by moving to the next regarded state.

As far as I know, this step is doable only in Mathematica, since Mathematica allows to program the dynamic stochastic economic model in a finite state space. It might not be possible in Matlab, since in Matlab one has to generate the state space, i.e. the grid for the model, which is then infinite.

⁷See for example Birk (2004).

vii. The final step is then to define the invariant distribution, either by inverting the transition probability matrix and then taking the vector which is associated with the eigenvalue of one, or taking the limit of the transition matrix, that is, iterating it. Usually after 50 to 100 iterations the stationary distribution is found. Both methods will yield the same invariant distribution function.

The invertibility of the transition matrix depends crucially on the finite state space and also on the property $F(\cdot) = 1$. That is, if we allow for a cumulative distribution function, i.e. for probabilities larger than one, the invertibility of the transition matrix might fail. In our macroeconomic models, however, this is usually not the case, since all endogenous variables can be expressed relative to output.

3.1 Calibration

Using the above model and simulation procedure, I compute and extensively analyze the numerical example for the labor market. A worker decides to accept a wage offer or to stay further on unemployed.

The model has been calibrated with the subsequent parameters.⁸ An employed worker can be laid off with probability $\lambda = 0.006$. Layoff costs are constant $K = 5$. The discount factor is $\beta = 0.95$.

The exogenous wage distribution $F(w)$ is Gaussian with mean 0.7 and variance 0.02. Wages are assumed to follow a Markov process. The wage for an employed worker stays constant with probability 0.98 and will be higher with probability 0.02.

The worker's skills change according to five skill levels evenly divided within the interval $h \in [1.0, 1.4]$. The skills of an employed worker remain with probability 0.95 the same, and she accumulates further on skills with probability 0.05.

Human capital depreciates when the worker is fired. When laid off, skills remain constant with probability 0.95 and devaluate with probability 0.05.

After the initial laid-off period, i.e. during unemployment, the worker's skills stay constant or depreciate. They devaluate twice as fast as they were accumulated, that is with probability 0.9 they remain the same and depreciate with probability 0.1.

The worker's disutility of searching for a job, c , and the probability of receiving a wage offer, π , depend positively on search effort, e , i.e.

$$\begin{aligned} c(e) &= 0.25 e \\ \pi(e) &= 0.5 e^{0.3}. \end{aligned}$$

The government redistributes income, via taxes and transfers, to workers to insure them against income fluctuations. For simplicity, I assume that an unemployed receives 60% of the last net income as unemployment benefits

$$b(I) = 0.6 I.$$

To distinguish between unemployed workers receiving benefits and those without, the government has a suitable earnings criterion. An unemployed worker, who quits the job or has been laid off, is entitled to unemployment compensation, when her last income is below 75% of the government's suitable earnings, $I_e(I) = I_u(I) = 0.75 I$. She is not eligible for unemployment benefits or her benefits will terminate, if she does not accept a job offer associated with earnings greater or equal to 75% of previous earnings or her past income was greater or equal than 75% of the government's suitable earnings criterion. The generated 25 last income levels fall into the interval $I \in [0, 1.5]$.

⁸The calibration parameters are in accordance with those chosen by Ljungqvist, Sargent (1998, 2005) or reflect data for the German labor market.

The tax rule used by the government has to be specified as well. Taxes are supposed to be proportional to the worker's net income with a non-progressive tax rate of $\tau = 0.3$. Thus, an employed worker receives an after tax income of $(1 - \tau)wh$, and an unemployed worker's net income is $(1 - \tau)b(I)$.

3.2 The calibrated equilibrium for the labor market

In the following, I will state the calibration results for the first exercise. Showing the optimal search intensities and the reservation wage functions for employed or unemployed workers seems not especially fruitful here and has been done, for example, by Ljungqvist, Sargent (1998, 2005). Therefore, I will jump straight to the discussion of model's results, where the homogenous labor force becomes heterogenous yet. Since each probability distribution \mathcal{P} over the state s converges to the stationary distribution $\psi(s)$ for *all* initial probability measures ψ_0 , the initial distributions can be read off from the initial transition matrix.

In the first example, the labor market is populated by unskilled workers with low previous earnings, i.e. $h = 1, I = 1$, thus $s = \{h_1, I_1\}$. For the first period, the calibrated transition matrix $\mathcal{P}(h_1, I_1)$ is

$$\mathcal{P}(h_1, I_1) = \begin{pmatrix} .55 & .45 \\ .11 & .89 \end{pmatrix}$$

and the stationary distribution for $t \rightarrow n < \infty$ is

$$\psi(h_1, I_1) = [.20 \quad .80].$$

For the first period, the impulse responses are—i.e. the transition matrix shows—that 55% are unemployed and 45% are employed currently, conditional on having been unemployed previously, and 11% are unemployed and 89% are employed currently, conditional on having been employed in the last period. After all the adjustment processes have taken place, and if we consider that the economy consists only of unskilled workers with low previous earnings, then for that case, the economy will have a stationary unemployment rate of 20% and an employment rate of 80%.

The adjustment processes of reaching the stationary steady state, i.e. the impulse-responses for $t = 2$, are the following

$$\mathcal{P}(h_1, I_1) = \begin{pmatrix} .35 & .65 \\ .16 & .84 \end{pmatrix}.$$

That is, in the second period and conditional on the previous unemployment of workers, the fraction of unemployed dropped from 55% to 35% and that for employed workers increased from 45% to 65%; and conditional on having been employed in the previous period, unemployment increased from 11% to 16% and employment decreased accordingly from 89% to 84%.

For the next period, $t = 3$, the impulse response is

$$\mathcal{P}(h_1, I_1) = \begin{pmatrix} .26 & .74 \\ .18 & .82 \end{pmatrix},$$

for $t = 4$ it is

$$\mathcal{P}(h_1, I_1) = \begin{pmatrix} .23 & .77 \\ .19 & .81 \end{pmatrix},$$

for $t = 5$ it is

$$\mathcal{P}(h_1, I_1) = \begin{pmatrix} .21 & .79 \\ .19 & .81 \end{pmatrix},$$

and for $t = 5$ it is

$$\mathcal{P}(h_1, I_1) = \begin{pmatrix} .20 & .80 \\ .20 & .80 \end{pmatrix}.$$

That is, in this first example, the economy reaches the stationary steady state after 5 periods with an unemployment rate of 20% and an employment rate of 80%.

In the second example, when workers are unskilled with past previous earnings, the calibrated transition matrix $\mathcal{P}(h_1, I_{25})$ is

$$\mathcal{P}(h_1, I_{25}) = \begin{pmatrix} .92 & .08 \\ .39 & .61 \end{pmatrix},$$

which induces an invariant distribution function of

$$\psi(h_1, I_{25}) = [.83 \ .17].$$

This stationary distribution shows that 83% of these workers will, most likely, be unemployed and 17% of them will be employed in the long-run invariant equilibrium.

The impulse-responses are the following

for $t = 1$

$$\mathcal{P}(h_1, I_1) = \begin{pmatrix} .88 & .12 \\ .60 & .40 \end{pmatrix},$$

for $t = 2$

$$\mathcal{P}(h_1, I_1) = \begin{pmatrix} .86 & .14 \\ .71 & .29 \end{pmatrix},$$

for $t = 3$

$$\mathcal{P}(h_1, I_1) = \begin{pmatrix} .84 & .16 \\ .76 & .24 \end{pmatrix},$$

until they will finally converge to the stationary distribution function $\psi(h_1, I_{25}) = [.83 \ .17]$.

In the last example, in which the economy is populated by skilled workers with high past earnings, the transition matrix $\mathcal{P}(h_5, I_{25})$ is

$$\mathcal{P}(h_5, I_{25}) = \begin{pmatrix} .89 & .11 \\ .15 & .84 \end{pmatrix},$$

which implies a stationary distribution function of

$$\psi(h_5, I_{25}) = [.59 \ .41].$$

This means when the economy has skilled workers with high previous incomes, 59% of them will be unemployed and 41% will be employed in the steady state.

4 Second Example

In the second example, I will confirm the above statements about the invertibility of the transition matrix \mathcal{P} , which means the transition probability matrix and the term $(I - \mathcal{P}')$ are the outcomes of the theoretical model and can be used as VARs by empirical researchers. I will show that the transition matrix is also a squared matrix and invertible and can, after some manipulations, according to Fernandez-Villaverde, Rubio-Ramirez and Sargent (2005), be used by VAR researchers to match the impulse responses of a VAR and the economic model. In this second exercise, a representative worker has to make more than one decision and adjudicates

not only upon employment, but also whether she will stay further on in her home country or moves to another country. The set up of this model changes only slightly compared to the first one.

The worker sees her new skill level at the beginning of a period, searches for a job in both countries, migrates or stays and is employed or unemployed in the next period. That is, at the beginning of each period t , the worker is unemployed and chooses search effort $e_{t,k} \geq 0$ to look for a job in the origin (source) or foreign (destination) country, $k \in \{o, f\}$. Let T_k be the fixed migration cost when moving. Search induces disutility $c(e_k)$, which is increasing in e_k , and may lead to a wage offer. With probability $\pi(e_{t,k})$, $\pi_{e_{t,k}} > 0$, the worker draws a wage offer in $t + 1$. The job offers come from the countries' time-invariant wage distributions $F_k(w_k) = \Pr(w_{k,t+1} \leq w_k)$ that differ in means and variances. The source country has a lower mean wage and smaller wage variances than the destination country. For each country, the set of possible wage values is denoted W_k , $W_k = \{w_{k,1}, \dots, w_{k,l}\}$. With the wage offer at hand, the worker can accept or reject the job offer in either country. Accepting a foreign offer implies the worker will migrate in $t + 1$ to the destination country and will be employed in period $t + 1$. Each wage follows a Markov process with stationary transition probability $G(w'_k | w_k) = \Pr(w_{k,t+1} \leq w'_k | w_{k,t} = w_k) > 0$, $\forall w_k, w'_k \in W_k$, independent of all other wage offers. Rejecting the wage offer means the worker is unemployed in period $t + 1$. With probability $(1 - \pi(e_{t,k}))$, the worker will not receive a wage offer at all.

Once the worker has quit the job or is unemployed, she can be entitled for social assistance. On the one hand, she will be eligible for unemployment benefits $b(I_k)$, if the government's suitable earning criterion $I_e(I_k)$ is higher than the worker's previous earned wage income $I_k (= w_k h)$. On the other hand, as long as the government's suitable earnings are lower than last earnings, no benefits are paid at all. Net benefits are $(1 - \tau)b(I_k)$ with τ as the tax rate. Due to simplicity, I assume that both countries have the same rules for the eligibility of benefits and that past earnings can be transferred to the other country without any institutional problems.

When the worker was previously employed, she can be laid off with probability $\lambda \in [0, 1]$ and is unemployed in period t . Let K be the layoff costs when being laid off or having quit the job. Laid off workers can qualify for unemployment compensation too, if the foregone earnings fall short of the government's suitable earnings criterion. That is, as long as $w_k h < I_u(I_k)$, the worker is eligible for benefits.

At each point in time, the worker's decision problem is described by an individual state vector $s_k \in S_k$ with $s_k = (h, I_k)$ and S_k is the individual state space with $S_k = H \times W_k$ for $H = \{h_1, \dots, h_m\}$ and $W_k = \{w_{k,1}, \dots, w_{k,l}\}$. On the basis of this state vector, the worker makes decisions for migration and employment.

After the characterization of the worker's environment, her functional equations will be defined. Let $V(h, w_k, I_k)$ be the value of the optimization problem for the worker with skill level h and wage w_k who was employed and earned income I_k in the previous period and who decides upon a country specific wage offer to migrate and quit the job. $V_b(h, I_k)$ is the value of the optimization problem for an unemployed worker with skill level h and country specific last earnings I_k who is entitled for unemployment compensation. $V_0(h)$ is the value for an unemployed worker who is not entitled to unemployment compensation. The Bellman equations for this sequential search and migration problem are:

$$V(h, w_k, I_k) = \max_{\text{accept, reject}} \{ \Omega(h, w_k), D(h, w_k, I_k)V_b(h, I_k) - T_k + (1 - D(h, w_k, I_k))V_0(h) - K \}, \quad (7)$$

$$\begin{aligned}
V_b(h, I_k) = & \max_{e_k} \left\{ -c(e_k) + (1 - \tau)b(I_k) + \beta \sum_{h'} \mu_u(h, h') \right. \\
& [(1 - \pi(e_k))V_b(h', I_k) + \pi(e_k)* \\
& \left. \left(\int_{w_k < I_u(I_k)/h'} \max_{\text{accept, reject}} \{ \Omega(h', w_k), V_b(h', I_k) \} dF_k(w_k) \right. \right. \\
& \left. \left. + \int_{w_k \geq I_u(I_k)/h'} \max_{\text{accept, reject}} \{ \Omega(h', w_k), V_0(h') \} dF_k(w_k) \right) \right\}, \tag{8}
\end{aligned}$$

$$\begin{aligned}
V_0(h) = & \max_{e_k} \left\{ -c(e_k) + \beta \sum_{h'} \mu_u(h, h') [(1 - \pi(e_k)) \right. \\
& \left. V_0(h') + \pi(e_k) \int \max_{\text{accept, reject}} \{ \Omega(h', w_k), V_0(h') \} dF_k(w_k) \right\}, \tag{9}
\end{aligned}$$

where

$$\begin{aligned}
\Omega(h, w_k) \equiv & (1 - \tau)w_k h + \beta \left[\lambda \sum_{h'} \mu_l(h, h') V_b(h', w_k h) - \lambda(K + T_k) + \right. \\
& \left. (1 - \lambda) \sum_{h'} \mu_e(h, h') \int V(w'_k, h', w_k h) dG(w'_k | w_k) \right], \\
D(h, w_k, I_k) = & \begin{cases} 1, & \text{if } w_k < I_e(I_k)/h, \\ 0, & \text{if } w_k \geq I_e(I_k)/h, \end{cases} \\
T_k = & \begin{cases} 1, & \text{if } \bar{w}_m(h, I_k) \geq \bar{w}_{b,k}(h, I_k), \\ 0, & \text{if } \bar{w}_m(h, I_k) < \bar{w}_{b,k}(h, I_k). \end{cases}
\end{aligned}$$

For each country $k \in \{o, f\}$, the intertemporal optimization problem (7) – (9) can be solved numerically. This gives functions, first, for the optimal search intensities $e_{b,k}^*(h, I_k)$ and the reservation wages $\bar{w}_{b,k}(h, I_k)$ for an unemployed worker who is eligible for unemployment compensation, second, for the optimal search intensities $e_{0,k}^*(h)$ and the reservation wages $\bar{w}_{0,k}(h)$ for an unemployed worker who is not entitled for benefits and, third, the reservation wages $\bar{w}_k(h, w_k, I_k)$ for an employed worker. The reservation migration wage $\bar{w}_m(h, I_k)$ for a migrating worker is defined as $\bar{w}_{m,k} \equiv \max\{\bar{w}_{b,o}, \bar{w}_{b,f}\}$. The functions $\bar{w}_{0,k} : S_k \rightarrow \mathbb{R}$, $\bar{w}_{b,k} : S_k \rightarrow \mathbb{R}$, $\bar{w}_{m,k} : S_k \rightarrow \mathbb{R}$ and $\bar{w}_k : S_k \rightarrow \mathbb{R}$ are optimal decision rules provided they are measurable, feasible and satisfy the functional equations as optimal value functions.

4.1 The stationary equilibrium for the labor market

With the optimal decision rules at hand, I define the stationary equilibrium for the source country, show examples for transition probability matrices and derive invariant distribution functions for various states. Even though workers look for jobs in two labor markets, the stationary equilibrium will be derived for the source country only.⁹ Therefore, $S_k \equiv S_o$, and for this section I just write S as the state space.

⁹The reasons for this is that, otherwise, I had to formulate the decision problem of a worker in the destination country as well. This would imply that I had to take her decision into account too, i.e. making it endogenous. After that, I should had set up a transition probability matrix, which had to hold for each country indicated by the subscript $k \in \{o, f\}$. In general, this is doable, but a bit more complicated and would not yield any new insights.

Since domestic workers will be heterogeneous in their individual state vectors, a way of describing the heterogeneity at a point in time is needed. Again, a probability measure will be defined on subsets of the individual state space to capture the heterogeneity. Let S be the finite set with $\{s_1, \dots, s_r\}$ and ψ be a probability measure on (S, \mathcal{S}) , where $S = H \times W$ and \mathcal{S} is the Borel σ -algebra. Thus, for $S \in \mathcal{S}$, $\psi(S)$ indicates the mass of workers, whose individual state vectors lie in S . Notice also that $F_k(0) = 0$ and $F_k(w_k) = 1$.

The next paragraph shows again how to construct the transition probabilities $\psi(\bar{w}'(s)|\bar{w}(s))$ and a Markov transition probability matrix from the decision rules $\bar{w}_{0,k}(s)$, $\bar{w}_{b,k}(s)$, $\bar{w}_k(s)$, $\bar{w}_m(s)$. Equipped with a well-defined transition function \mathcal{P} , a probability measure ψ defined on (S, \mathcal{S}) is stationary.

Definition 4 *A stationary recursive equilibrium for the labor market of this economy are reservation wage functions $(\bar{w}_{0,k}(s), \bar{w}_{b,k}(s), \bar{w}_k(s), \bar{w}_m(s))$ and distribution functions $\psi(s)$ such that*

- (1) $\bar{w}_{0,k}, \bar{w}_{b,k}, \bar{w}_k$ solve the individual's optimization problem (6)–(8); $\bar{w}_{m,k} \equiv \max\{\bar{w}_{b,o}, \bar{w}_{b,f}\}$
- (2) an n -dimensional vector $s \in R^n$ records the possible values of the state of the system;
- (3) an $(n \times 1)$ vector ψ_0 records the probabilities of being in each state i at time 0 with
 - (a) $\psi_{0i}(S) = \Pr[w_0(s) = \bar{w}_i(s)]$,
 - (b) $\psi_0(S) = \sum_{i=1}^n \psi_{0i}(s) = 1$;
- (4) an $n \times n$ transition matrix $\mathcal{P}(s, S)$ records the probabilities of moving from one value of the state to another in one period;
- (5) $\psi(s)$ is time-invariant.

Once again, I compute equilibria for the source country's labor market and restrict the analysis to situations, in which a worker is eligible for unemployment compensation only. The reservation wage functions $\bar{w}_{b,o}(s)$ and $\bar{w}_m(s)$ remain. With these functions at hand, the partial equilibrium analysis can continue.

The following characterizes a worker's initial probability distribution over states, i.e. situations where the worker migrates or stays and accepts a wage offer or remains unemployed. That is, with the reservation wages for employment and migration, $\bar{w}_{b,o}(s)$ and $\bar{w}_m(s)$, respectively, a worker can be in one of four states

$$\psi_{0i}(s) = \begin{cases} F_f(\bar{w}_m(s)) * F_o(\bar{w}_{b,o}(s)), & w_f < \bar{w}_m(s), \bar{w}_{b,o}(s) > w_o, \\ [1 - F_f(\bar{w}_m(s))] * F_o(\bar{w}_{b,o}(s)), & w_f > \bar{w}_m(s), \bar{w}_{b,o}(s) > w_o, \\ F_f(\bar{w}_m(s)) * [1 - F_o(\bar{w}_{b,o}(s))], & w_f < \bar{w}_m(s), \bar{w}_{b,o}(s) \leq w_o, \\ [1 - F_f(\bar{w}_m(s))] * [1 - F_o(\bar{w}_{b,o}(s))], & w_f > \bar{w}_m(s), \bar{w}_{b,o}(s) \leq w_o. \end{cases} \quad (10)$$

The decision rules define the worker's unconditional probabilities $\psi_{0i}(s)$ at time 0 for $i = 1, \dots, 4$. In the first state, the worker is unemployed and stays in the source country, whereas $F_o(\bar{w}_{b,o}(s))$ reflects the unconditional probability of being unemployed, which depends on the reservation wage of being eligible for unemployment benefits, and $F_f(\bar{w}_m(s))$ is defined as the unconditional probability of staying in the source country, which depends on the reservation wage for migration. The second state reflects the unconditional probability of being unemployed, but moving to the destination country. The third state is defined as the unconditional probability

of staying in the source country and being employed. And finally, the fourth state is defined as the unconditional probability of being employed in the destination country.

In the next step, I use these unconditional probabilities to define the Markov transition matrix, which shows then the conditional transition probabilities of moving from one state to another.

Proposition 5 *Let $\mathcal{P}_{ij} = \mathcal{P}(s_i, \{s_j\})$ be the conditional probability of being currently in state i and move to state j in the next period, then the transition probabilities of the states are*

$$\mathcal{P}_{ij,o} = \Pr[w_{t+1}(s) = \bar{w}_j(s) | w_t(s) = \bar{w}_i(s)] =$$

$$\begin{pmatrix} F_m[\pi F_o + (1 - \pi)] & (1 - F_m)[\pi F_o + (1 - \pi)] & F_m\pi(1 - F_o) & (1 - F_m)\pi(1 - F_o) \\ \gamma[\pi F_f + (1 - \pi)] & (1 - \gamma)[\pi F_f + (1 - \pi)] & \gamma\pi(1 - F_f) & (1 - \gamma)\pi(1 - F_f) \\ F_m[\lambda + (1 - \lambda)F_o] & (1 - F_m)[\lambda + (1 - \lambda)F_o] & F_m(1 - \lambda)(1 - F_o) & (1 - F_m)(1 - \lambda)(1 - F_o) \\ \gamma[\lambda + (1 - \lambda)F_f] & (1 - \gamma)[\lambda + (1 - \lambda)F_f] & \gamma(1 - \lambda)(1 - F_f) & (1 - \gamma)(1 - \lambda)(1 - F_f) \end{pmatrix} \quad (11)$$

Proof. Since $\mathcal{P}_{ij} \geq 0$ and $\sum_{j=1}^n \mathcal{P}_{ij} = 1$ for $i = 1, \dots, n$, \mathcal{P} is an $n \times n$ Markov matrix. ■

Notice that I have used the abbreviations of $F_o \equiv F_o(\bar{w}_{b,o}(s))$ as the unconditional probability of being unemployed in the source country, $F_f \equiv F_f(\bar{w}_{b,f}(s))$ as the unconditional probability of being unemployed in the foreign country, $(1 - F_m(\bar{w}_m(s)))$ as the unconditional probability of migrating to the destination country and $F_m \equiv F_f(\bar{w}_m(s))$ as the unconditional probability of staying in the source country. Furthermore, $\pi \equiv \pi(e_f^*(s))$ is the endogenous job offer probability.

The first row reflects all future states conditional on being currently unemployed in the source country. Especially, $\mathcal{P}_{11,o}$ is the conditional probability of being unemployed in the next period in the source country, $\mathcal{P}_{12,o}$ is the conditional probability of being unemployed in the next period in the destination country, where γ is the exogenous probability of staying in the destination country. $\mathcal{P}_{13,o}$ is the conditional probability of being employed in the source country in the future, and $\mathcal{P}_{14,o}$ is the conditional probability of being employed in the destination country in the future. The second row $i = 2$ states the transition probabilities for the next period conditional on being currently unemployed in the destination country, $i = 3$ defines the possible states in the next period conditional on being currently employed in the source country and $i = 4$ shows the transitions states for the next period conditional on being employed in the destination country currently.

With the transition function $\mathcal{P}(s, S)$ at hand, the equilibrium of the source country is stationary and described by the invariant distribution $\psi(s)$, which is shown in the next proposition:

Proposition 6 *If $\psi(s)$ satisfies*

$$(I - \mathcal{P}')\psi = 0, \quad (12)$$

the equilibrium is stationary and characterized by the invariant distribution function $\psi(s)$.

Proof. (12) determines $\psi(s)$ as an eigenvector associated with a unit eigenvalue of \mathcal{P}' . That is, the fact that \mathcal{P} is a stochastic matrix, i.e., it has nonnegative elements and satisfies $\sum_j P_{ij} = 1$ for all i , guarantees that \mathcal{P} has at least one unit eigenvalue, and there is at least one eigenvector that satisfies equation (12). ■

4.2 Calibration

I follow the same simulation procedure as described in Section 3, and I take nearly the same parameter values for the second model as I used in the first model except for the following. Migration costs depend on the worker's decision to migrate or to stay. If she migrates, migration costs are $T_f = 12$ and otherwise $T_o = 0$. The exogenous probability of returning from the destination to the source country ($1 - \gamma$) is 0.05, a value similar to empirical studies. The exogenous wage distributions $F_k(w_k)$, $k \in \{o, f\}$, are Gaussian distributions with mean 0.5 and variance 0.01 for the source country and with mean 0.7 and variance 0.02 for the destination country.

4.3 Calibrated stationary equilibrium for the labor market

I neglect to state the reservation wages for a worker who decides about migrating to another country or staying in her home country, which I did in my (2005) paper. Here I will discuss how to derive the transition probability matrices taking into account that the reservation wages for employment and migration of a worker have been simulated. Using these optimal decision policies for an individual migrant, the stationary equilibrium of the labor market for the source country will be calibrated.

In the first example, the source country is populated by unskilled workers with low previous earnings, $s = \{h_1, I_1\}$. For the first period, the calibrated transition matrix $\mathcal{P}(h_1, I_1)$ is

$$\mathcal{P}(h_1, I_1) = \begin{pmatrix} .06 & .54 & .04 & .36 \\ .52 & .03 & .43 & .02 \\ .02 & .19 & .08 & .71 \\ .10 & .01 & .85 & .04 \end{pmatrix}$$

and the stationary distribution is

$$\psi(h_1, I_1) = [.13 \quad .15 \quad .39 \quad .33].$$

The impulse responses shown in the transition matrix are for the first row, which expresses the condition of being currently unemployed in the source country, the probability of being further on unemployed in the source country in the next period is $\mathcal{P}_{11} = .06$, the probability of being unemployed in the destination country in the next period is $\mathcal{P}_{12} = .54$, the probability of being employed in the source country is $\mathcal{P}_{13} = .04$ and finally the probability of being employed in the destination country in the next period is $\mathcal{P}_{14} = .36$, and so forth for the remaining rows. Finally the economy, which is populated by unskilled workers with low previous incomes, converges to its stationary steady state, $t \rightarrow n < \infty$, in which 13% of the workers will probably stay unemployed in the source country (they are called 'unemployed stayers'), 15% want to migrate and will most likely be unemployed in the destination country ('unemployed movers'), 39% will stay employed in the source country ('employed stayers'), and 33% are employed movers. That is, ψ describes the stationary distribution for this economy.

Here are further examples for the impulse-responses towards the equilibrium. For $t = 2$, the impulse-responses are

$$\mathcal{P}(h_1, I_1) = \begin{pmatrix} .32 & .06 & .54 & .08 \\ .06 & .36 & .09 & .49 \\ .17 & .04 & .69 & .10 \\ .03 & .21 & .11 & .64 \end{pmatrix},$$

for $t = 3$

$$\mathcal{P}(h_1, I_1) = \begin{pmatrix} .07 & .28 & .15 & .51 \\ .24 & .06 & .59 & .11 \\ .05 & .23 & .16 & .56 \\ .18 & .05 & .65 & .12 \end{pmatrix},$$

for $t = 4$

$$\mathcal{P}(h_1, I_1) = \begin{pmatrix} .22 & .08 & .57 & .15 \\ .07 & .25 & .18 & .51 \\ .18 & .07 & .59 & .16 \\ .06 & .22 & .18 & .53 \end{pmatrix},$$

and so forth until the economy reaches finally the stationary distribution $\psi(h_1, I_1)$.

In the second example, the source country is populated by unskilled workers with high previous earnings, i.e. $h = 1, I = 25$, thus $s = \{h_1, I_{25}\}$. The calibrated transition matrix $\mathcal{P}(h_1, I_{25})$ is

$$\mathcal{P}(h_1, I_{25}) = \begin{pmatrix} .36 & .58 & .02 & .04 \\ .88 & .05 & .07 & .01 \\ .19 & .30 & .19 & .31 \\ .37 & .02 & .58 & .03 \end{pmatrix}$$

and the stationary distribution is

$$\psi(h_1, I_{25}) = [.52 \quad .35 \quad .08 \quad .05].$$

For this state with unskilled workers with high past incomes, 52% of them will, most likely, be unemployed stayers, 35% are unemployed movers, 8% are employed stayers, and 5% are employed movers.

In the third example, the source country is populated by middle skilled workers with high previous earnings, i.e. $h = 3, I = 25$, thus $s = \{h_3, I_{25}\}$. The calibrated transition matrix $\mathcal{P}(h_3, I_{25})$ is

$$\mathcal{P}(h_3, I_{25}) = \begin{pmatrix} .14 & .80 & .01 & .05 \\ .85 & .04 & .10 & .01 \\ .07 & .42 & .07 & .43 \\ .15 & .01 & .80 & .04 \end{pmatrix}$$

and the stationary distribution is

$$\psi(h_3, I_{25}) = [.41 \quad .40 \quad .11 \quad .08].$$

For this state with middle skilled workers with high past incomes, 41% of them will, most likely, be unemployed stayers, 40% are unemployed movers, 11% are employed stayers, and 8% are employed movers.

In the last example, the source country is populated by skilled workers with high previous earnings, i.e. $h = 5, I = 25$, thus $s = \{h_5, I_{25}\}$. The calibrated transition matrix $\mathcal{P}(h_5, I_{25})$ is

$$\mathcal{P}(h_5, I_{25}) = \begin{pmatrix} .09 & .84 & .01 & .06 \\ .84 & .04 & .11 & .01 \\ .05 & .44 & .05 & .46 \\ .10 & .01 & .85 & .04 \end{pmatrix},$$

and the stationary distribution is

$$\psi(h_5, I_{25}) = [.38 \quad .40 \quad .13 \quad .09].$$

For this state with skilled workers with high past incomes, 38% of them will, most likely, be unemployed stayers, 40% are unemployed movers, 13% are employed stayers, and 9% are employed movers.

5 Conclusion

In the paper, I showed that the transition matrix of a stochastic dynamic macro model can be defined as a squared matrix which is then, in connection with an identity matrix and the finite state space, invertible. The calibrated impulse-responses of the model can now be compared with those generated by empirical researchers.

6 Literature

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