

COMPARISON OF FISSION PRODUCT INVENTORIES COMPUTED
USING MATRIX EXPONENTIAL AND G-FACTOR TECHNIQUES

A Thesis

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Master of Science

in

The Department of Nuclear Engineering

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Dedicated to my family

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ABSTRACT

Fission product data has been assembled for 192 fission product chains from mass 66 to mass 172 consisting of 798 isotopes. This data includes half life and branching factor values in addition to 3 group fission yields from Th-232, U-233, U-235, U-238, Pu-239, and Pu-241. This data was incorporated into program MIKU written by R. E. Miles which is capable of computing activities in problems involving radioactive buildup, decay, and mass transfer. This program uses either the matrix exponential method or a G-factor method developed by R. E. Miles. A modeling of the decay chain data described above was chosen which is compatible with both the matrix exponential and G-factor methods. Therefore program MIKU can be used to compute fission product activities using either method.

The matrix exponential and G-factor methods have been analyzed and compared theoretically. Both methods are capable of handling unlimited forward branching. However, in addition the matrix exponential method is capable of handling problems involving reverse branching. The G-factor method is more efficient computationally due to the usage of recurrence relations. The G-factor method has been found to be from 2 to 5 times faster depending on

the length of the time intervals used than usage of the matrix exponential method option.

Program MIKU has been used to analyze and compare results from several sample problems using both the matrix exponential and G-factor methods. These problems included runs for several isotope chains for different reactor power operating times, and various fuel enrichments. The results obtained have been found to be in perfect agreement to at least 10 significant figures. This amazing agreement is due to the accuracy of the two methods and to the usage of triple precision arithmetic available on LSU's IBM 3033 computer.

MIKU is capable at present of handling all isobaric fission product isotopes with half life greater than 1 sec, since all fission yield data, branching factors, and half life data has been included. The program input is simple and easy to use. Execution is accomplished using a simple CLIST command.

CHAPTER ONE

Introduction

The main objectives of this thesis has been to assemble fission product decay data in a form compatible with program MIKU written by R. E. Miles to analyze and compare theoretically the matrix exponential and G-factor methods for use in treating problems involving radioactive buildup, decay, and mass transfer and to test program MIKU using the above data for several sample problems.

The data assembled consist of fission product decay chain data from mass 66 to mass 172 for isotopes with half lives greater than 1 sec. Since there was no attempt in this thesis to assemble activation, actinide, transuranium, gamma ray spectra or other similar data in this thesis the discussion will center around fission buildup and decay analysis with some limited discussion of reactor accident analysis.

The linear differential equations for buildup and decay of fission products in a reactor are well known but it becomes difficult to obtain their solution as the length of the decay chain increases. If there are only two or three members in a decay chain then their solution can easily be obtained by direct integration. For more complex problems where there are seven or eight members

in a decay chain in which various branching possibilities are available, it is very tedious and virtually impossible to solve them by direct integration. Several methods exist for obtaining accurate solutions of these differential equations which avoids these difficulties as well as the problem of singularities that occurs when the depletion rate constants of two or more nuclides in the same chain are equal. These methods include the matrix exponential, finite difference, and the G-factor method recently developed by R. E. Miles (Ref. 1-4).

This thesis will compare the matrix exponential and G-factor methods for the solution of fission product buildup and decay problems. These methods both have a much wider application but the discussion of this thesis is limited to the application of the fission product chain data assembled as a part of this thesis.

The matrix exponential method provides a very general approach to radioactive buildup and decay problems. It can also be used to calculate the integrated fission product releases from a reactor containment building to the environment and the activity absorbed on filters are washed from the containment building by sprays. Thus it is a useful technique for dose projections to be used in emergency response planning. This method can also accommodate reverse branching which is important in fuel burnup computation. The method makes use of matrix theory

and operators to simplify the mathematics involved and reduces the problem to setting up the differential equations or alternatively the supermatrix A. In Chapter Two, several decay chains are considered to explain clearly how supermatrix A is formed.

The G-factor method approaches the problem in a different way. Simple exponentials are computed only once and then recurrence relations are used to determine higher order terms or E-factors. The G-factor consists of various E-factor terms and can be shown to be related to the probability of atom transfers between various nuclides in a decay chain. Computationally the method is extremely efficient due to the usage of these recurrence relations. This method while allowing unlimited forward branching is not capable of treating reverse branching.

These methods can be applied to a variety of problems. Cases like simple radioactive decay transformations, calculation of routine and accidental releases from nuclear reactors, buildup and decay of isotopes in long chains with unlimited forward branching, and for the matrix exponential method reverse branching. Singularities present no problem for either method.

CHAPTER TWO

Matrix Exponential Method

This is a powerful and elegant method for handling problems involving buildup, decay, transmutation and mass transport of radionuclides. As the name implies it uses matrix theory and solution of matrix differential equations to simplify the mathematics involved (Ref. 5). In this section the method will be developed utilizing a reactor accident analysis model. However, it should be understood that the method can be applied to other problems involving radioactive decay, buildup, transmutation and mass transport with little modification.

In a reactor accident analysis model it is simple to include the presence of cleanup filter in the containment building and in case of accidents, the leakage of arbitrary isotope of any chain from the containment building to the environment as a function of time.

There are two assumptions in this model:

- a) The removal rate of the clean up filter in the containment building and the leakage rate of isotope from the containment building to the environment, are constant during short intervals of time.

- b) The gas inside the containment building is well mixed.

Initial source for the isotopes is taken to be leakage from reactor vessel to the containment building. Natural deposition of gas borne isotopes on the internal surface of the containment building is neglected in order to avoid unnecessary complexity.

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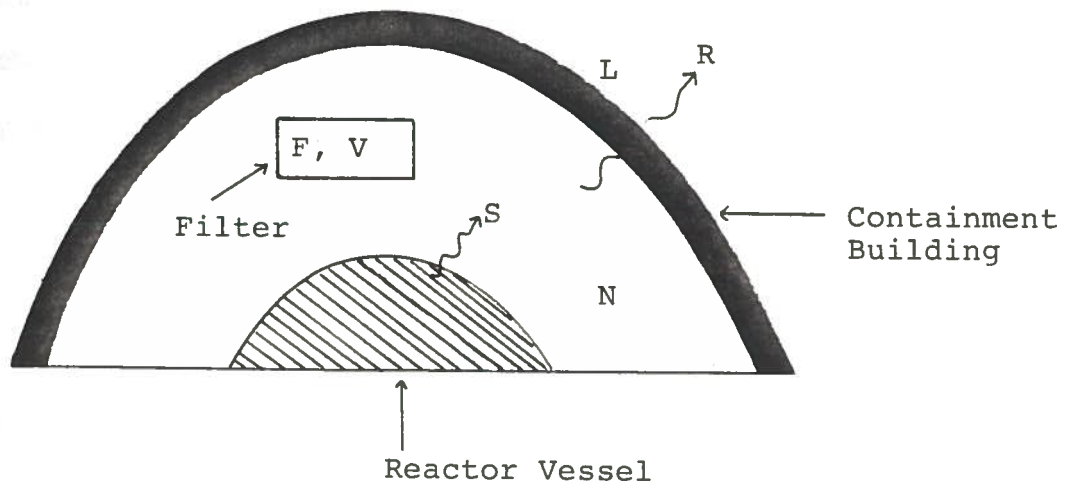
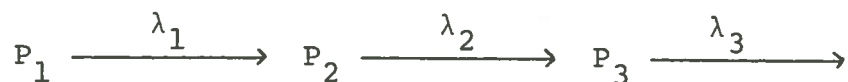


Fig. 1

Reactor Accident Analysis Model

Mathematical Basis

Let us consider a system shown in Fig. 1. The system consists of a reactor vessel and a filter system inside the containment building. Before we set up the equations in matrix form, it is easier to understand them term by term for three isotopes of a chain and then generalize it for any member of long decay chain using matrix theory. We note that noble gases are not filtered by the clean up system. As an illustration consider the three member decay chain



where P_2 is a noble gas.

We assume the following:

N_1, N_2, N_3 = Amount of isotopes $P_1, P_2,$ and P_3
in the containment building at time t .

V_1, V_2, V_3 = Filter clean up rate for $P_1, P_2,$ and
 P_3 .

L_1, L_2, L_3 = Leakage rate of containment building
to environment for $P_1, P_2,$ and P_3 .

F_1, F_2, F_3 = Amount of $P_1, P_2,$ and P_3 absorbed on
the filter.

$\lambda_1, \lambda_2, \lambda_3$ = Decay constants of $P_1, P_2,$ and P_3 .

The equations governing amount of any isotope in the
containment building and on the filter at any time t are:

$$\frac{dN_1}{dt} = - (\lambda_1 + V_1 + L_1) N_1(t) + S_1 \quad (1)$$

$$\frac{dN_2}{dt} = \lambda_1 N_1(t) - (\lambda_2 + V_2 + L_2) N_2(t) + \lambda_1 F_1(t) + S_2 \quad (2)$$

$$\frac{dN_3}{dt} = \lambda_2 N_2(t) - (\lambda_3 + V_3 + L_3) N_3(t) + S_3 \quad (3)$$

and

$$\frac{dF_1}{dt} = V_1 N_1(t) - \lambda_1 F_1 \quad (4)$$

$$\frac{dF_2}{dt} = 0 \quad (5)$$

$$\frac{dF_3}{dt} = V_3 N_3(t) - \lambda_3 F_3(t) \quad (6)$$

All the terms are obvious except for $\lambda_1 F_1(t)$ in Eq. (2). This term accounts for the fact that isotope P_1 which is absorbed in the filter can decay to noble gas contribute to N_2 in the containment building. In Equation (5), rate of change of F_2 with time is zero because noble gas is not absorbed in the filter.

These six equations can be put in matrix form illustrated in Fig. 2.

$$\begin{bmatrix} \frac{dN_1}{dt} \\ \frac{dN_2}{dt} \\ \frac{dN_3}{dt} \\ \frac{dF_1}{dt} \\ \frac{dF_2}{dt} \\ \frac{dF_3}{dt} \end{bmatrix} = \begin{bmatrix} -(\lambda_1 + V_1 + L_1) & 0 & 0 & 0 & 0 & 0 \\ \lambda_1 & -(\lambda_2 + V_2 + L_2) & 0 & 0 & 0 & 0 \\ 0 & \lambda_2 & -(\lambda_3 + V_3 + L_3) & 0 & 0 & 0 \\ V_1 & 0 & 0 & -\lambda_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & V_3 & 0 & 0 & -\lambda_3 \end{bmatrix} \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ F_1 \\ F_2 \\ F_3 \end{bmatrix} + \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{7}$$

Fig. 2. Matrix Form for the Six Equations. (Supermatrix A is 6 x 6 Matrix)

Matrix Formulation for Long Arbitrary Chain

Equations (1) to (6) are easily modified for long decay chains including branching factors. An isotope of a decay chain can get contributions from many other members of the same chain due to branching. To take branching into account, the decay constants are multiplied by corresponding branching factors and the products are called $\bar{\lambda}_{ij}$ and they are used to replace decay constants in the previous example. Again the supermatrix A can be determined by first writing the appropriate differential equations for the decay chain and then rearranging the terms so that the differential equations can be expressed using matrix notation. An alternative approach which breaks the supermatrix up into 4 submatrices will now be presented.

In this alternative approach another set of constants λ_{ij}^* are introduced to simplify the formulation. These are taken to be negative and defined as the product of decay constants and corresponding branching factors for only those isotopes which decay to noble gases. The generalized equations are:

$$\begin{aligned} \frac{dN_1}{dt} = & (\bar{\lambda}_{21}N_2 + \bar{\lambda}_{31}N_3 + \dots + \bar{\lambda}_{51}N_5 \dots) - (\lambda_{12}N_1 + \lambda_{13}N_1 \dots) \\ & - (\lambda_{21}^*F_2 + \lambda_{31}^*F_3 + \dots) - V_1N_1 - L_1N_1 + S_1 \end{aligned}$$

(8)

$$\begin{aligned} \frac{dN_2}{dt} = & (\bar{\lambda}_{12} N_1 + \bar{\lambda}_{32} N_3 + \dots) - (\bar{\lambda}_{21} N_2 + \bar{\lambda}_{23} N_2 + \dots) \\ & - (\lambda_{12}^* F_1 + \lambda_{32}^* F_3 + \dots) \\ & - V_2 N_2 - L_2 N_2 + S_2 \end{aligned} \quad (9)$$

$$\frac{dF_1}{dt} = V_1 N_1 - \lambda_{12} F_1 - \lambda_{13} F_1 \dots \lambda_{14} F_1 \dots \quad (10)$$

$$\frac{dF_2}{dt} = V_2 N_2 - \lambda_{21} F_2 - \lambda_{23} F_2 \dots \lambda_{24} F_2 \dots \quad (11)$$

where λ_{ij} is product of decay constants and branching factors for those isotopes which do not decay to noble gas.

A little consideration of equations (8) to (11) show that they can be put in matrix form with the following changes:

[N], [F], and [s] are taken as vectors, the elements of which are the values of the individual members of the chain.

$[L]$ and $[V]$ are the diagonal leak and filter clean up rate matrices. The diagonal elements of these matrices represent the leakage and clean up rates of each individual member of chain.

$[\bar{\lambda}]$ is taken as decay chain matrix. The diagonal elements of which are positive and the negative off diagonal elements are the products of branching factors and decay constants. $[\bar{\lambda}]$ and $[\lambda^*]$ become matrices.

We note that dimension of matrices are $n \times n$ and number of elements of vector $[N]$, $[F]$, and $[s]$ are n where n is the number of isotopes in a given chain.

With these changes, the equations become

$$\frac{d}{dt} \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ \vdots \\ \vdots \\ \vdots \end{bmatrix} = - \Lambda \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ \vdots \\ \vdots \\ \vdots \end{bmatrix} - \lambda^* \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ \vdots \\ \vdots \\ \vdots \end{bmatrix} + \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ \vdots \\ \vdots \\ \vdots \end{bmatrix} \quad (12)$$

and

$$\frac{d}{dt} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ \vdots \\ \vdots \end{bmatrix} = \begin{bmatrix} V_1 & & & & \\ & V_2 & & & \\ & & V_3 & & \\ & & & \ddots & \\ & & & & \ddots \end{bmatrix} \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ \vdots \\ \vdots \end{bmatrix} - \lambda \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ \vdots \\ \vdots \end{bmatrix} \quad (13)$$

where

$$[\Lambda] = [\bar{\lambda}] + [V] + [L]$$

From now onwards [] sign won't be used to indicate a matrix for convenience. The Eq. (12) and (13) in matrix form are then

$$\frac{dN}{dt} = -\Lambda N - \lambda^* F + S \quad (14)$$

$$\frac{dF}{dt} = V N - \lambda F \quad (15)$$

Note that from definition of $\bar{\lambda}$, λ and λ^* a simple relation exist among them

$$\bar{\lambda} = \lambda + \lambda^*$$

The solutions to Eq. (14) and (15) are obtained in the following manner (Ref. 1)

Define

$$X = \begin{pmatrix} N \\ F \end{pmatrix}, \quad s = \begin{pmatrix} S \\ 0 \end{pmatrix}$$

then

and supermatrix $A = \begin{pmatrix} -\Lambda & -\lambda^* \\ \text{---} & \text{---} \\ V & -\lambda \end{pmatrix}$

Defin

Now $AX + s = \begin{pmatrix} -\Lambda & -\lambda^* \\ V & -\lambda \end{pmatrix} \begin{pmatrix} N \\ F \end{pmatrix} + \begin{pmatrix} S \\ 0 \end{pmatrix}$

then

$$= \begin{pmatrix} -\Lambda N - \lambda^* F \\ VN - \lambda F \end{pmatrix} + \begin{pmatrix} S \\ 0 \end{pmatrix} = \begin{pmatrix} -\Lambda N + \lambda^* F + S \\ VN - \lambda F \end{pmatrix}$$

so

which

$$\frac{dx}{dt} = \frac{d}{dt} \begin{pmatrix} N \\ F \end{pmatrix} = \begin{pmatrix} -\Lambda N - \lambda^* F + S \\ VN - \lambda F \end{pmatrix}$$

then

$$\Rightarrow \frac{dx}{dt} = AX + s \quad (16)$$

The

N.

We assume matrices A and s are constant over time interval $(0, t)$.

Integrated release from containment building is denoted by $R(t)$

$$R(t) = \int_0^t L(t')N(t')dt'$$

The leakage rate L can be taken as a constant or some average value can be assigned to it.

$$\bar{L} = 1/2[L(0) + L(t)]$$

then

$$R(t) = \bar{L} \int_0^t N(t') dt' \quad (17)$$

Define

$$Y(t) = \begin{pmatrix} R(t) \\ 0 \end{pmatrix}, \quad B = \begin{pmatrix} \bar{L} & 0 \\ 0 & 0 \end{pmatrix}$$

then Eq. (17) becomes

$$Y(t) = B \int_0^t X(t') dt' \quad (18)$$

Def.

which is clear from

then

$$\begin{pmatrix} R(t) \\ 0 \end{pmatrix} = \begin{pmatrix} \bar{L} & 0 \\ 0 & 0 \end{pmatrix} \int_0^t \begin{pmatrix} N \\ F \end{pmatrix} dt'$$

The solutions of Eq. (16) and (18) gives us the value for N, F, and the integrated release. Solution of Eq. (16) is given by:

$$X(t) = e^{At} [X(0) + \int_0^t X^{-1}(t') s(t') dt']$$

where

$$X(t') = e^{At'}$$

$$X(t) = e^{At} X(0) + [e^{At} \int_0^t dt' e^{-At'}] s$$

$$= e^{At} X(0) + e^{At} \left[\int_0^t dt' e^{-At'} \right] s$$

From

$$= e^{At} X(0) - e^{At} A^{-1} [e^{-At} - 1]s$$

$$= e^{At} X(0) - A^{-1} [e^{At} e^{-At} - e^{At}]s$$

so

$$= e^{At} X(0) - A^{-1} [1 - e^{At}]s$$

$$= e^{At} X(0) + A^{-1} [e^{At} - 1]s \quad (19)$$

Define $D(c) = c^{-1}[\exp(c) - 1]$

then $D(At) = A^{-1} t^{-1} [\exp(At) - 1]$ ($c = At$)

or

$$tD(At) = A^{-1} [e^{At} - 1]$$

then

$$e^{At} = tD(At) A + 1$$

so that

$$e^{At} X(0) = tD(At)A X(0) + X(0)$$

Eq. (19) becomes

$$X(t) = X(0) + tD(At) [A X(0) + s] \quad (20)$$

Now we solve Eq. (18) $Y(t) = B \int_0^t dt' X(t')$

From (19) we have

$$X(t') = e^{At'} X(0) + A^{-1} [e^{At'} - 1]s$$

Hence,
so

$$\begin{aligned} Y(t) &= B \int_0^t \left\{ dt' e^{At'} X(0) + [A^{-1} e^{At'} - A^{-1}]s \right\} \\ &= B [A^{-1}(e^{At} - 1) X(0) + A^{-1} A^{-1} [e^{At} - 1]s \\ &\quad - A^{-1} ts] \end{aligned}$$

Let us take the terms inside the bracket one by one

$$A^{-1}(e^{At} - 1) X(0) = tD(At) X(0)$$

Now

for second term we note that

$$D(At) = A^{-1} t^{-1} (e^{At} - 1)$$

or

$$tD(At) = A^{-1} (e^{At} - 1)$$

Multiply by A^{-1} on both sides, we get

$$t A^{-1} D(At) = A^{-1} A^{-1} (e^{At} - 1)$$

$$t t t^{-1} A^{-1} D(At) = A^{-1} A^{-1} (e^{At} - 1)s$$

$$t^2 c^{-1} D(At)s = A^{-1} A^{-1} (e^{At} - 1)s$$

and

$$A^{-1} t s = t A^{-1} s = t t t^{-1} A^{-1} s = t^2 c^{-1} s$$

Hence,

$$\begin{aligned} Y(t) &= B [tD(At) X(0) + t^2 \{c^{-1}D(c) - c^{-1}\} s] \\ &= B [tD(At) X(0) + t^2 z(At)s] \end{aligned} \quad (21)$$

where

$$cZ(c) = D(c) - 1$$

$$\therefore Z(c) = c^{-1}D(c) - c^{-1}$$

Now matrix operators $D(c)$ and $Z(c)$ are given by

$$\begin{aligned} D(c) &= c^{-1} [\exp(c) - 1] = \sum_{n=0}^{\infty} \frac{c^n}{(n+1)!} \\ Z(c) &= c^{-1} [D(c) - 1] = \sum_{n=0}^{\infty} \frac{c^n}{(n+2)!} \end{aligned} \quad (22)$$

Direct evaluation of $D(c)$ and $Z(c)$ are difficult computationally because C is a matrix

Take $H = 2^{-P}C$

where

$$p \text{ is determined by } |H| < 1/2$$

or

$$p > \ln(|C_{ij}|^2) / 2 \ln 2$$

We can approximate $D(H)$ and $Z(H)$ by taking only finite number of terms M

$$D^M(H) = \sum_{n=0}^M \frac{H^n}{(n+1)!}$$

$$Z^M(H) = \sum_{n=0}^M \frac{H^n}{(n+2)!}$$

M is determined such that excluded terms are less than some ϵ .

$$\frac{|H|^{M+1}}{(M+2)!} < \frac{1}{2^{M+1} (M+2)!} < \epsilon$$

Recur upwards by powers of 2 in H to find $D(c)$ and $Z(c)$ where $c = 2^P H$ and recursion relations are

$$D(2^{P+1}H) = D(2^P H) \left[1 + \left(\frac{1}{2}\right) (2^P H) D(2^P H) \right] \quad (23)$$

$$Z(2^{P+1}H) = \left(\frac{1}{2}\right) Z(2^P H) + \frac{1}{4} [D(2^P H)]^2 \quad (24)$$

These recursion relations are very useful in computation. With these relations the solution to Eq. 16 and Eq. 18 are obtained in a manner which makes computation easier for computer. We now discuss some decay chains to illustrate how to set up the various matrices.

The differential equations for decay chain (mass number 85)

and

$\frac{dF_1}{dt}$

$\frac{dF_2}{dt}$

$\frac{dF_3}{dt}$

are:

$$\frac{dA_1}{dt} = -(\lambda_1 + v_1 + L_1) A_1(t) + S_1$$

$$\frac{dA_2}{dt} = -(\lambda_2 + v_2 + L_2) A_2(t) + S_2$$

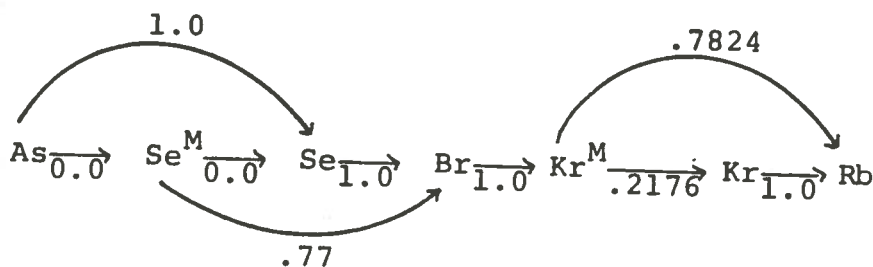
$$\frac{dA_3}{dt} = -(\lambda_3 + v_3 + L_3) A_3(t) + \lambda_1 A_1(t) + \lambda_1 F_1(t) + S_3$$

$$\frac{dA_4}{dt} = -(\lambda_4 + v_4 + L_4) A_4(t) + \lambda_3 A_3(t) + \lambda_2 A_2(t) + \lambda_3 F_3(t)$$

$$+ \lambda_2 F_2(t) + S_4$$

$$\frac{dA_5}{dt} = -(\lambda_5 + L_5) A_5(t) + \lambda_4 A_4(t) + \lambda_4 F_4(t) + S_5$$

$$\frac{dA_6}{dt} = -(\lambda_6 + L_6) A_6(t) + \lambda_5 A_5(t) + S_6$$



$$\frac{dA_7}{dt} = -(\lambda_7 + V_7 + L_7) A_7(t) + \lambda_5 A_5(t) + \lambda_6 A_6(t) + S_7$$

and

$$\frac{dF_1}{dt} = V_1 A_1(t) - \lambda_1 F_1(t)$$

$$\frac{dF_2}{dt} = V_2 A_2(t) - \lambda_2 F_2(t)$$

$$\frac{dF_3}{dt} = V_3 A_3(t) - \lambda_3 F_3(t) + \lambda_1 F_1(t)$$

$$\frac{dF_4}{dt} = V_4 A_4(t) - \lambda_4 F_4(t) + \lambda_2 F_2(t) + \lambda_3 F_3(t)$$

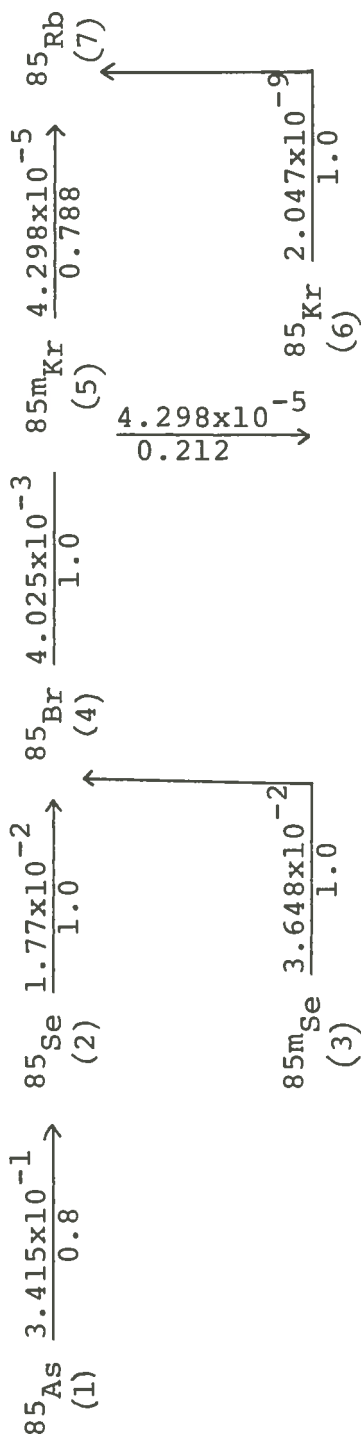
$$\frac{dF_5}{dt} = 0$$

$$\frac{dF_6}{dt} = 0$$

$$\frac{dF_7}{dt} = V_7 A_7(t) - \lambda_7 F_7(t)$$

These equations can be easily put in matrix form as was done in previous examples giving us supermatrix A. The supermatrix A can be obtained without writing the differential equations. Consider Fig. 3 which shows decay chain for mass number 85. The numbers above the

arrow represent decay constants and below the arrow represent branching factors. Since there are seven members in the decay chain, the matrices $\bar{\lambda}$, λ , and λ^* will be seven by seven matrices.



All decay constants are in sec^{-1}

Fig. 3
Decay Chain for Mass 85

$$\bar{\lambda} = \begin{bmatrix} 3.415 \times 10^{-1} & 0 & 0 & 0 & 0 & 0 & 0 \\ -2.72 \times 10^{-1} & 1.77 \times 10^{-2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3.648 \times 10^{-2} & 0 & 0 & 0 & 0 \\ 0 & -1.77 \times 10^{-2} & -3.468 \times 10^{-2} & 4.025 \times 10^{-3} & 0 & 0 & 0 \\ 0 & 0 & 0 & -4.025 \times 10^{-3} & 4.298 \times 10^{-5} & 0 & 0 \\ 0 & 0 & 0 & 0 & -9.122 \times 10^{-6} & 2.047 \times 10^{-9} & 0 \\ 0 & 0 & 0 & 0 & -3.387 \times 10^{-5} & -2.047 \times 10^{-9} & 0 \end{bmatrix}$$

Fig. 3(a). Matrix $\bar{\lambda}$ for Decay Chain (Mass Number 85)

Some of the elements of the above matrix are calculated for illustration

$$\bar{\lambda}_{21} = 3.415 \times 10^{-1} \times 0.8 = 2.72 \times 10^{-1}$$

Now $-\lambda_{21}$ means that this term is for decay of first isotope (As-85) to the second (Se-85).

$$\bar{\lambda}_{65} = 4.298 \times 10^{-5} \times 0.212 = 9.122 \times 10^{-6}$$

Again the negative sign has the same meaning.

$\bar{\lambda}_{77}$ is taken to be zero since it is the term for stable isotope.

To set up the matrix λ^* notice that fifth and sixth members are noble gases. Now fourth member (Br-85) decays to fifth and fifth member decays to sixth.

So,

$$\lambda_{54}^* = - (4.025 \times 10^{-3} \times 1) = - 4.025 \times 10^{-3}$$

$$\lambda_{65}^* = - (4.298 \times 10^{-5} \times 0.212) = - 9.122 \times 10^{-6}$$

and all other elements of λ^* are zero.

Matrix λ can easily be obtained from matrix $\bar{\lambda}$ by the relation

$$\bar{\lambda} - \lambda^* = \lambda$$

$$\lambda^* = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -4.025 \times 10^{-3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -9.122 \times 10^{-6} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Fig. 3(b). Matrix λ^* for Decay Chain (Mass Number 85)

After
matrix
rate
chain

which

$$\lambda = \begin{bmatrix} 3.415 \times 10^{-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2.72 \times 10^{-1} & 1.77 \times 10^{-2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3.648 \times 10^{-2} & 0 & 0 & 0 & 0 & 0 \\ 0 & -1.77 \times 10^{-2} & -3.648 \times 10^{-2} & 4.025 \times 10^{-3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4.298 \times 10^{-5} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2.047 \times 10^{-9} & 0 & 0 \\ 0 & 0 & 0 & 0 & -3.387 \times 10^{-5} & -2.047 \times 10^{-9} & 0 & 0 \end{bmatrix}$$

Fig. 3(c). Matrix λ for Decay Chain (Mass Number 85)

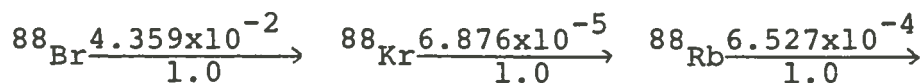
After obtaining $\bar{\lambda}$, λ^* , and λ we can set up the super matrix A by taking some reasonable values for the cleanup rate and the leakage rate for each member of the decay chain. Then A will be

$$A = \begin{pmatrix} -\Lambda & | & -\lambda^* \\ \hline -V & | & -\lambda \end{pmatrix}, \quad \Lambda = \bar{\lambda} + V + L$$

which will be fourteen by fourteen matrix for this chain.

Decay Chain for Mass Number 88

We set up the various matrices for the following decay chain



(All decay constants are in sec^{-1}).

Since there are three members in the decay chain the matrices $\bar{\lambda}$, λ , and λ^* will be 3 x 3 matrices. Also no branching is taking place so for this chain the matrices become quite simple. They are

$$\bar{\lambda} = \begin{pmatrix} 4.359 \times 10^{-2} & 0 & 0 \\ -4.359 \times 10^{-2} & 6.876 \times 10^{-5} & 0 \\ 0 & -6.876 \times 10^{-5} & 6.527 \times 10^{-4} \end{pmatrix}$$

$$\lambda = \begin{pmatrix} 4.359 \times 10^{-2} & 0 & 0 \\ 0 & 6.876 \times 10^{-5} & 0 \\ 0 & -6.876 \times 10^{-5} & 6.527 \times 10^{-4} \end{pmatrix}$$

and

$$\lambda^* = \begin{pmatrix} 0 & 0 & 0 \\ -4.359 \times 10^{-2} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

The supermatrix A for this chain will be then a 6 x 6 matrix.

CHAPTER THREE

G-Factor Method

This technique is developed by Dr. Robert Miles, Louisiana State University, for handling radioactive decay, buildup and mass transfer problems. The procedure is simplified by introducing the concepts of path specific probability function. General solutions are obtained which include the branching factors and independent production of nuclides. Use of recurrence relations for exponential terms makes the computation fast and efficient. Singularities are also treated easily by this technique which will be discussed later in the section. The advantage of G-factor technique lies in the fact that it is easy to understand and apply to problems since very simple concepts are involved.

Let us first consider a generalized decay chain as shown in Fig. 4. We define the following:

A_i = the number of atoms of the i th nuclide present at time t .

u_i = the total removal constant of the i th nuclide.

k_{ij} = the transfer rate constant from the i th to the j th nuclide ($i > j$, reverse branching, which means branching from daughter to ancestor, is not allowed).

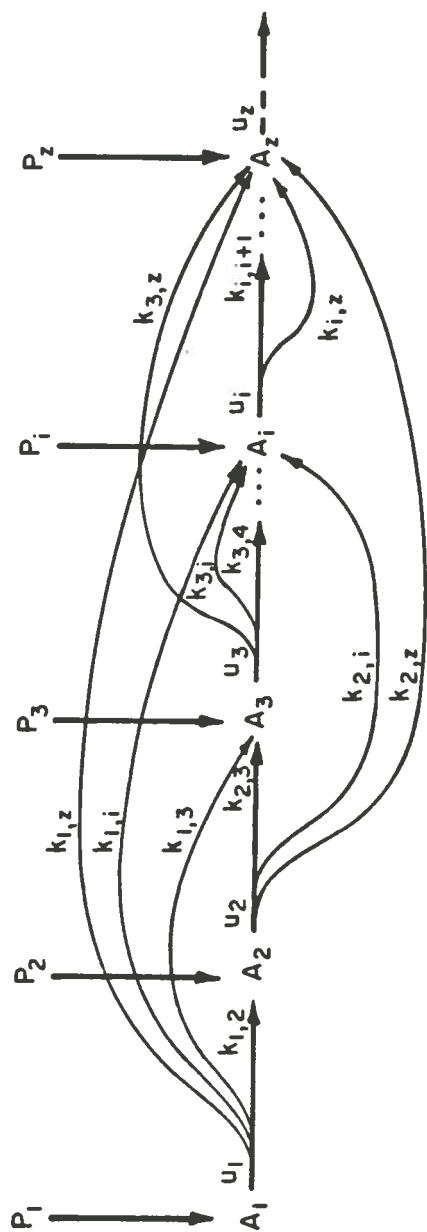


Fig. 4. Arbitrary Decay Chain with Branching

There are several terms which contribute to the total removal constant u_i . They are

- a) Radioactive decay constant
- b) Removal rate due to nuclear reactions
- c) Mass transfer rate for the i th nuclide

The transfer rate constant k_{ij} is present because of various reasons. It can be just the material transfer rate from i^{th} to j^{th} nuclide or the production rate of j^{th} nuclide due to nuclear reaction involving i^{th} nuclide. Also it can be product of decay constant for the i^{th} nuclide and the branching fraction from the i^{th} to j^{th} nuclide.

The analysis of radioactive decay chains is simplified by dividing it into two parts. One in which we study the decay chain when there is no independent production of any nuclide of the chain and other where we take into account independent production.

Case 1. Decay Chain with no Independent Production

In this case the system of differential equations governing the number of atoms of each nuclide as a function of time are:

$$\frac{dA_1(t)}{dt} = -u_1 A_1(t) \quad (25)$$

$$\frac{dA_2(t)}{dt} = k_{12} A_1(t) - u_2 A_2(t) \quad (26)$$

$$\frac{dA_3(t)}{dt} = k_{13} A_1(t) + k_{23} A_2(t) - u_3 A_3(t) \quad (27)$$

$$\frac{dA_z(t)}{dt} = \sum_{m=1}^{z-1} k_{mz} A_m(t) - u_z A_z(t) \quad (28)$$

We define $E_i = e^{-u_i t}$ $i = 1, 2, 3, \dots, z$ (29)

and solve the first three equations. By direct integration we get

$$A_1(t) = A_1^0 E_1 \quad (30)$$

$$A_2(t) = k_{12} A_1^0 \left\{ E_2 \int_0^t \frac{E_1 dt'}{E_2} \right\} + A_2^0 E_2 \quad (31)$$

$$A_3(t) = k_{13} A_1^0 \left\{ E_3 \int_0^t \frac{E_1 dt'}{E_3} \right\} + k_{23} \left[E_3 \int_0^t \frac{k_{12} A_1^0}{E_3} \right.$$

$$\left. \left\{ E_2 \int_0^t \frac{E_1 dt'}{E_2} \right\} dt'' \right] + k_{23} A_2^0 \left\{ E_3 \int_0^t \frac{E_2 dt'}{E_3} \right\}$$

$$+ A_3^0 E_3 \quad (32)$$

where

A_i^0 = initial number of atoms (at $t = 0$)

Equations (30), (31), and (32) can be put in a simple form by defining E factors and using exponential recurrence relations.

$$E_{ij} = E_j \int_0^t \frac{E_i dt'}{E_j} = \frac{E_i - E_j}{u_j - u_i} \quad u_i \neq u_j, \quad j = 1, 2, 3, \dots, z \quad (33)$$

$$E_{ijk} = E_k \int_0^t \frac{E_{ij} dt'}{E_k} = \frac{E_{ij} - E_{jk}}{u_k - u_i}$$

$k = 1, 2, 3, \dots, z$ and

$$u_i \neq u_j \neq u_k \quad (34)$$

...

$$E_{ijk\dots yz} = E_z \int_0^t \frac{E_{ijk\dots y} dt'}{E_z} = \frac{E_{ijk\dots y} - E_{jk\dots yz}}{u_z - u_i}$$

$$\text{for } u_i \neq u_j \neq u_k \neq \dots \neq u_z \quad (35)$$

Now equations (30), (31), and (32) can be written as

$$A_1(t) = A_1^0 E_1 \quad (36)$$

$$A_2(t) = k_{12} A_1^0 E_{12} + A_2^0 E_2 \quad (37)$$

$$\begin{aligned}
 A_3(t) = & k_{12} k_{23} A_1^0 E_{123} + k_{13} A_1^0 E_{13} + k_{23} A_2^0 E_{23} \\
 & + A_3^0 E_3
 \end{aligned} \tag{38}$$

The above equations indicate a symmetry which we can exploit to write the solution for the fourth nuclide in the chain.

$$\begin{aligned}
 A_4(t) = & A_1^0 \left\{ k_{12} k_{23} k_{34} E_{1234} + k_{12} k_{24} E_{124} + k_{13} k_{34} E_{134} \right. \\
 & \left. + k_{14} E_{14} \right\} + A_2^0 \left\{ k_{23} k_{34} E_{234} + k_{24} E_{24} \right\} + A_3^0 k_{34} E_{34} \\
 & + A_4^0 E_4
 \end{aligned} \tag{39}$$

Define

$$G_{ij} = k_{ij} E_{ij}$$

$$G_{ijk} = k_{ij} k_{jk} E_{ijk}$$

$$G_{ijk---yz} = (k_{ij} k_{jk} \dots k_{yz}) (E_{ijk---yz}) \tag{40}$$

We get

$$A_1(t) = A_1^0 E_1 \tag{41}$$

$$A_2(t) = A_1^0 G_{12} + A_2^0 E_2 \quad (42)$$

$$A_3(t) = A_1^0 \{ G_{123} + G_{13} \} + A_2^0 G_{23} + A_3^0 E_3 \quad (43)$$

$$A_4(t) = A_1^0 \{ G_{1234} + G_{124} + G_{134} + G_{14} \} + A_2^0 \{ G_{234} + G_{24} \} \\ + A_3^0 G_{34} + A_4^0 E_4 \quad (44)$$

Again we see a symmetry in the equations and we can write the number of atoms of any nuclide in the chain. A closer look at the G-factors indicate that if interpreted correctly, the method of solving all the differential equations reduce to tracing the possible branching paths from i^{th} to j^{th} nuclide.

Identifying $G_{ijk---yz}$ as path function which represents the probability that an atom of the i^{th} type will be transformed into z atom by a specified path $jk---y$ in time t , considerable simplification in mathematics is achieved. Note also that function $G_{ijk---yz}$ occurs in the solution of z^{th} differential equation.

There are two ways to calculate the total number of possible paths from nuclide i to nuclide z . Note that there are $(z-i-1)$ nuclides which occur between i and j^{th} nuclide and there are only two possibilities for any nuclide between i and j , either to be included in the

branching path or excluded. Considering $(i + 1)^{\text{th}}$ to $(j - 1)$ nuclide together the total number of possible paths are obtained simply by multiplication.

$$(2)_{i+1} (2)_{i+2} (2)_{i+3} \dots (2)_{z-1} = (2)^{z-i-1}$$

Same result will be obtained by considering the number of ways in which intervening nuclides can be included or excluded in branching paths.

There are $z-i-1 C_0$ ways which exclude all nuclides between i and j .

There are $z-i-1 C_1$ ways which include only one nuclide between i and z .

$z-i-1 C_2$ ways which include two nuclides between i and z .

⋮

$z-i-1 C_{z-i-1}$ ways which include all $(z-i-1)$ nuclides between i and z .

So the total number of ways is just the sum of all the terms above

$$z-i-1 C_0 + z-i-1 C_1 + z-i-1 C_2 + \dots + z-i-1 C_{z-i-1} = \sum_{k=0}^{z-i-1}$$

$$C_k^{z-i-1}$$

which is

$$1 + (z-i-1) + \frac{(z-i-1)(z-i-2)}{2!} + \frac{(z-i-1)(z-i-2)(z-i-3)}{3!} + \dots$$

$$= (1 + 1)^{z-i-1} = 2^{z-i-1}$$

The total number of terms required in the solution for any nuclide z is given by

$$1 + \sum_{i=1}^{z-1} (2)^{z-i-1}$$

The reason for adding one is due to the fact that we have term $A_z^0 E_z$ which is present because of initial atoms of z^{th} nuclide.

The path specific probability function G defined can be calculated by the simple recurrence relation

$$G_{ijk---yz} = \frac{k_{yz} G_{ijk---y} - k_{ij} G_{jk---yz}}{u_z - u_i}$$

or from the E factor

$$G_{ijk---yz} = k_{ij} k_{jk---} k_{yz} E_{ijk---yz}$$

It is clear from Eq. (34) that for $u_z = u_i$, the E factors are not defined. An alternate expression can be developed for such cases. These will be considered in the next section.

The general solution to the zth differential equation can now be written as $A_z^0 E_z$ plus the sum of all the path specific probability functions multiplied by initial amount of the originating nuclide. Thus, the solution of zth differential equation reduces to the problem of determining all possible paths to the zth nuclide. For further simplicity we introduce cumulative transfer function $G(t, i \rightarrow z)$ which is defined as the probability that an atom of the ith nuclide will be transformed into nuclide z through all possible paths in time t. From definition it is clear that cumulative transfer function $G(t, i \rightarrow z)$ is the sum of all the path specific probability functions for transformation of nuclide i to z and can be represented by:

$$G(t, i \rightarrow z) = \sum_{\text{all } G} G_{ijk \dots z}$$

The solution in terms of $G(t, i \rightarrow z)$ becomes

$$A_z(t) = \sum_{i=1}^{z-1} A_i^0 G(t, i \rightarrow z) + A_z^0 E_z \quad (45)$$

We note that in many cases branching factors are zero which makes probability function $G_{ijk---yz}$ zero for the simple reason that $G_{ijk---yz}$ depends on k_{ij} which is clear from Equation (40). Hence if a particular branching fraction k_{ij} is zero then there is no need of computing G factors which include k_{ij} .

Case 2. Independent Production of Radionuclide Included

Assuming initial amount of each nuclide to be zero, the general differential equations, including independent production are:

$$\frac{dA_1(t)}{dt} = P_1 - u_1 A_1(t)$$

$$\frac{dA_2(t)}{dt} = k_{12} A_1(t) + P_2 - u_2 A_2(t)$$

$$\frac{dA_3(t)}{dt} = k_{13} A_1(t) + k_{23} A_2(t) + P_3 - u_3 A_3(t)$$

⋮
⋮
⋮

$$\frac{dA_z(t)}{dt} = \sum_{m=1}^{z-1} k_{mz} A_m(t) + P_z - u_z A_z(t) \quad (46)$$

The solution of these differential equations are developed in similar manner. We define

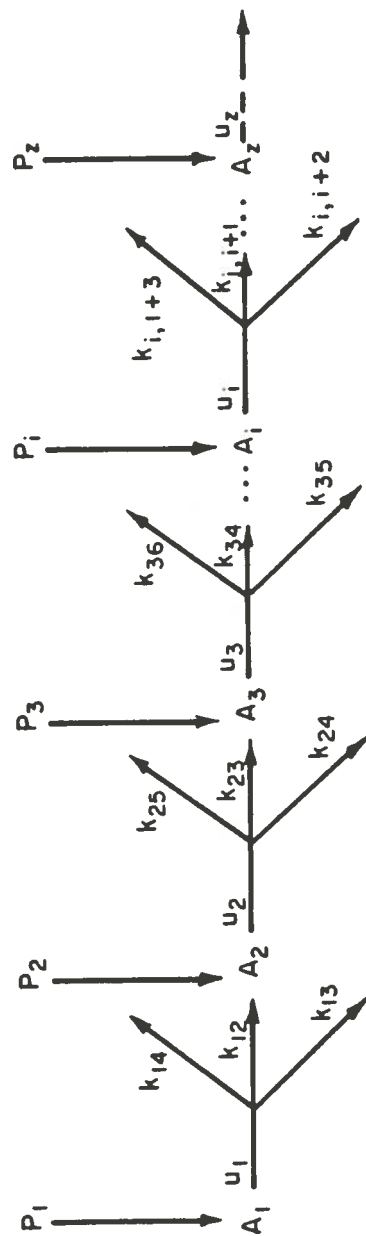


Fig. 5. Another Way of Representing Arbitrary Decay Chain with Independent Production.

$$E_i = e^{-u_i t}$$

$$E_{ij} = E_j \int_0^t \frac{E_i dt}{E_j} = \frac{E_i - E_j}{u_j - u_i} \quad j = 1, 2, \dots, z$$

$$u_i \neq u_j \quad (47)$$

$$E_{ijk---yz} = E_z \int_0^t \frac{E_{ijk---y} dt}{E_z} = \frac{E_{ijk---y} - E_{jk---yz}}{u_z - u_i}$$

$$\text{for } u_i \neq u_j \neq u_k \neq u_z$$

and take $u_0 = 0$

Further, we define new E terms. They are

$$E_{oi} = E_i \int_0^t \frac{E_o dt}{E_i} = \frac{E_o - E_i}{u_i - u_0} \quad i = 1, 2, \dots, z, u_i \neq 0$$

$$E_{oij} = E_j \int_0^t \frac{E_{oi} dt}{E_j} = \frac{E_{oi} - E_{ij}}{u_j - u_0} \quad j = 1, 2, 3, \dots, z,$$

$$u_j \neq 0$$

$$E_{oij---yz} = E_z \int_0^t \frac{E_{oij---y} dt}{E_z} = \frac{E_{oij---y} - E_{ijk---z}}{u_z - u_0} \quad (48)$$

$$z = 1, 2, 3, \dots, z, u_2 \neq 0.$$

similarly we define path specific probability function. Subscript zero is used to distinguish from the previous case (with no independent production)

$$G_{oij} = k_{ij} E_{oij}$$

$$G_{oijk} = k_{ij} k_{jk} E_{oijk}$$

·
·
·

$$G_{oijk---yz} = (k_{ij} k_{jk---k_{yz}}) E_{oijk---yz}$$

As in the previous case $G_{oijk---yz}$ represents the probability that an atom of the i th nuclide will be produced and then transformed into the nuclide specified by the last subscript by the path indicated by the intervening subscripts in time t . Using the recurrence relation

$$G_{oijk---yz} = \frac{k_{yz} G_{oijk---y} - G_{ijk---yz}}{u_z - u_o}$$

the function G can be calculated again in similar manner. The cumulative transfer probability function in this case is written $G(t, o i \rightarrow z)$ with the same definition as before. The general solution (without initial atoms) is

$$A_z(t) = \sum_{i=1}^{z-1} P_i G(t, o i \rightarrow z) + P_z E_{oz} \quad (49)$$

If we include the initial atoms present with independent production, the general solution is then sum of Eq. (45) and Eq. (49)

$$A_z(t) = \sum_{i=1}^{z-1} \left\{ A_i^0 G(t, i \rightarrow z) + P_i G(t, \rightarrow oi \ z) \right\} + A_z^0 E_z + P_z E_{Oz} \quad (50)$$

Treatment of singularities. Two classes of singularities exist which must be treated separately. They are encountered when

a) The total removal constant for the first and last nuclide in a given transformation path containing at least three nuclides are equal. There should be at least one intermediate nuclide with unique removal constant.

b) The total removal constant for all nuclides in a given transformation path are equal.

These singularities can be treated by noting the symmetry of E factors which were developed earlier. E factors are symmetric functions of their arguments. This can be shown easily.

$$E_{123} = \frac{e^{-u_1 t}}{(u_2 - u_1)(u_3 - u_1)} + \frac{e^{-u_2 t}}{(u_1 - u_2)(u_3 - u_2)} + \frac{e^{-u_3 t}}{(u_1 - u_3)(u_2 - u_3)}$$

which implies

$$E_{123} = E_{132} = E_{213} = E_{231} = E_{312} = E_{321}$$

The equivalence of E factors can be utilized in the treatment of singularities.

$$E_{ijk} = E_{ikj} = \frac{E_{ik} - E_{kj}}{u_j - u_i} = \frac{E_{ik} - E_{jk}}{u_j - u_i} \quad \text{for } u_i = u_k \neq u_j$$

$$E_{ijkl} = E_{ijlk} = \frac{E_{ijl} - E_{jlk}}{u_k - u_i} = \frac{E_{ijl} - E_{jkl}}{u_k - u_i} \quad \text{for } u_i = u_j \neq u_k$$

$$E_{ijkl} = E_{iklj} = \frac{E_{ikl} - E_{jkl}}{u_j - u_i} \quad \text{for } u_i = u_l \neq u_j$$

.

and

$$E_{ijk\cdots yz} = E_{ik\cdots yzj} = \frac{E_{ik\cdots yz} - E_{jk\cdots yz}}{u_j - u_i}$$

$$\text{for } u_i = u_z \neq u_j$$

where j is any integer between i and z.

For the second class of singularities, we integrate directly.

$$E_{ij} = E_j \int_0^t \frac{E_i dt}{E_j} = t E_j = t E_i \quad \text{for } u_i = u_j$$

$$E_{ijk} = E_k \int_0^t \frac{t E_i}{E_k} = \frac{t^2 E_k}{2!} = \frac{t^2 E_i}{2!} \quad \text{for } u_i = u_j = u_k$$

$$E_{ijkl} = E_i \int_0^t \frac{t^2 E_i dt}{2! E_i} = \frac{t^3 E_i}{3!} \quad \text{for } u_i = u_j = u_k$$

⋮

$$E_{ijk\dots z} = E_z \int_0^t \frac{t^{(z-i-1)} E_i dt}{(z-i-1)! E_z} = \frac{t^{(z-1)} E_i}{(z-i)!}$$

$$\text{for } u_i = u_j = u_k = \dots u_z$$

These expressions easily remove the singularities when encountered. Some useful relations involving derivatives and integrals of E factors can be developed.

Define

$$D_i = \frac{dE_i}{dt} = -u_i E_i$$

and

$$I_i = \int_0^t E_i dt = \frac{1-E_i}{u_i}$$

The recurrence relation are

$$D_{ij} = \frac{dE_{ij}}{dt} = \frac{D_i - D_j}{u_j - u_i} \quad \text{for } u_i \neq u_j$$

$$D_{ijk} = \frac{dE_{ijk}}{dt} = \begin{cases} \frac{D_{ij} - D_{jk}}{u_k - u_i} & \text{for } u_i \neq u_k \\ \frac{D_{ik} - D_{jk}}{u_j - u_i} & \text{for } u_i = u_k \neq u_j \end{cases}$$

and

$$I_{ij} = \int_0^t E_{ij} dt = \frac{I_i - I_j}{u_j - u_i} \quad \text{for } u_i \neq u_j$$

$$I_{ijk} = \int_0^t E_{ijk} dt = \begin{cases} \frac{I_{ij} - I_{jk}}{u_k - u_i} & \text{for } u_i \neq u_k \\ \frac{I_{ik} - I_{jk}}{u_j - u_i} & \text{for } u_i = u_k \neq u_j \end{cases}$$

Also for the class two singularities

$$D_{ij} = \frac{dE_{ij}}{dt} = E_i (1 - u_i t) \quad \text{for } u_i = u_j$$

$$D_{ijk} = \frac{dE_{ijk}}{dt} = t E_i \left(1 - \frac{u_i t}{2}\right) \quad \text{for } u_i = u_j = u_k$$

⋮

$$D_{ijk\dots z} = \frac{dE_{ijk\dots z}}{dt} = \frac{t^{(z-i-1)} E_i}{(z-i-1)!} E_i \left(1 - \frac{u_i t}{z-1}\right)$$

$$\text{for } u_i = u_j = u_k = u_z.$$

and

$$I_{ij} = \int_0^t E_{ij} dt = \int_0^t t E_i dt = \frac{1}{u_i} (E_{0i} - t E_i)$$

$$u_i = u_j$$

$$I_{ijk} = \int_0^t E_{ijk} dt = \frac{2!}{u_i^2} E_{oi} - E_i \left(\frac{t^2}{u_i} + \frac{2t}{u_i} \right)$$

$$u_i = u_j = u_k.$$

⋮

$$I_{ijk---z} = \frac{(z-i)!}{u_i^{z-1}} E_{oi} - E_i \left\{ \sum_{y=0}^{z-i-1} \frac{(z-i)! t^{z-i-y}}{(z-i-y)! u_i^{y+1}} \right\}$$

$$\text{for } u_i = u_j = u_k = u_z$$

Table 3.1

List of the Path Specific Probability Functions, G for Radionuclide Chains Containing from Z=1 to Z=6 Nuclides

Z=1	Z=2	Z=3	Z=4	Z=5	Z=6	
None	G_{12}	G_{123}	G_{1234}	G_{12345}	G_{123456}	G_{156}
			G_{134}	G_{1235}	G_{12346}	G_{146}
			G_{134}	G_{1245}	G_{12356}	G_{136}
			G_{234}	G_{1345}	G_{12456}	G_{126}
			G_{14}	G_{2345}	G_{13456}	G_{256}
			G_{24}	G_{125}	G_{23456}	G_{246}
			G_{34}	G_{135}	G_{1236}	G_{236}
			G_{145}	G_{1246}	G_{356}	
			G_{235}	G_{1346}	G_{346}	
			G_{245}	G_{1256}	G_{16}	
			G_{345}	G_{1356}	G_{26}	
			G_{15}	G_{1456}	G_{36}	
			G_{25}	G_{2346}	G_{46}	
			G_{35}	G_{2356}	G_{56}	
G_{45}	G_{2456}					

CHAPTER FOUR

MIKU Program

The block diagram of program MIKU which was written by R. E. Miles (Ref. 7) is shown in Fig. 6. This program consists of the various subroutines indicated in Fig. 6. Some of the subroutines like GSPEC do not exist at the present time but will be developed later. GSPEC will make it possible to compute the gamma spectrum as a function of time.

Input instructions for MIKU are also given and they are self-explanatory. Decay chains and isotope I.D. numbers are given in Appendix A. Fission yield data for all isotopes is given in Appendix B. This program takes little CPU time and input instructions are such that a beginner can run the program without any difficulty.

Some of the subroutines are briefly discussed.

Power - this subroutine determines fission rate

CDATA - calls input

DECAY - this subroutine converts half life into decay constant

UTEST - this subroutine multiplies decay constant by branching factors. Also it tests for singularity.

SCALER - this subroutine multiplies a matrix by a scaler

MULTI - multiplies two matrices

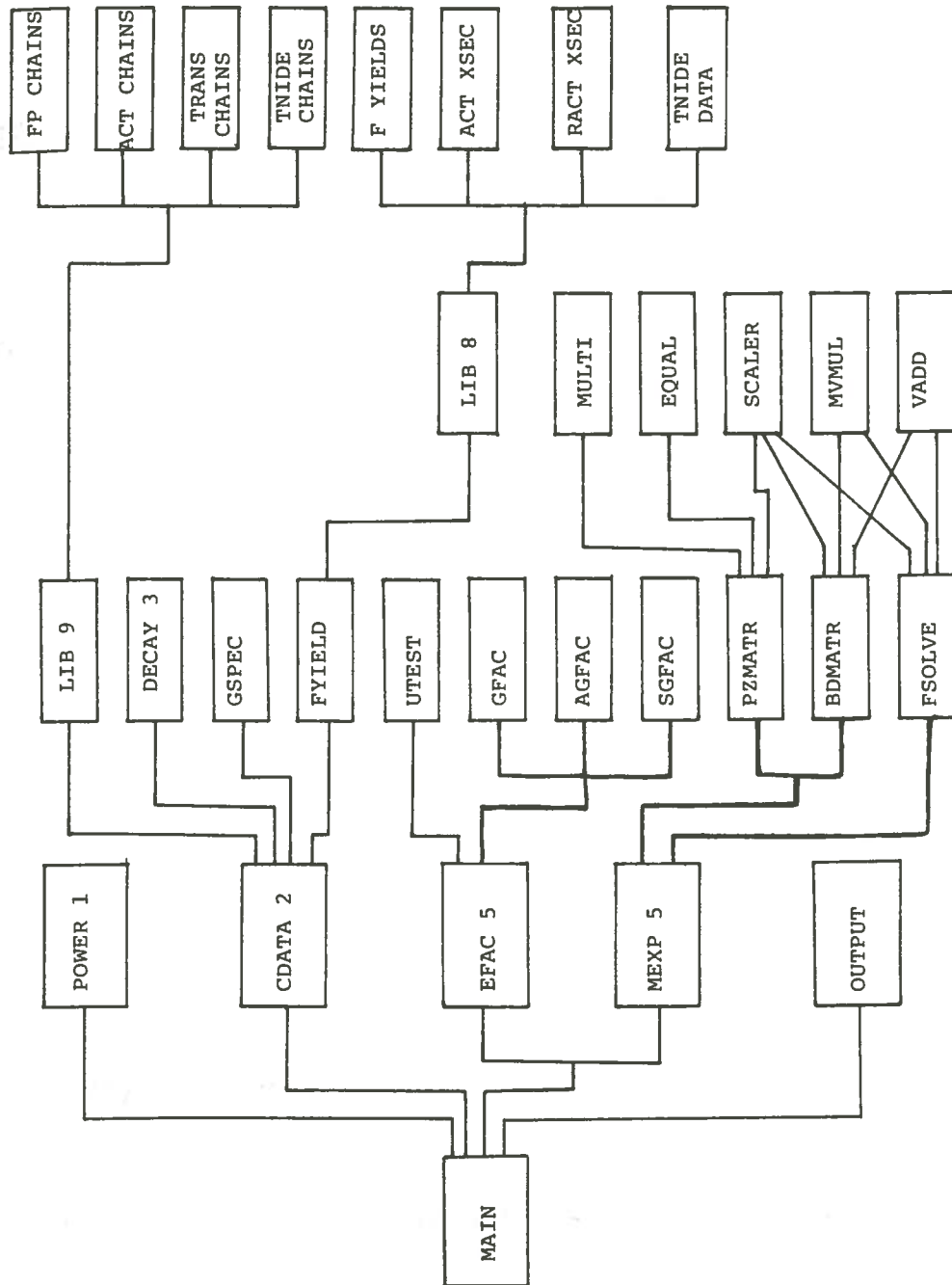
EQUAL - sets one matrix equal to other

MVMUL - this subroutine gives the product of matrix
times a vector

VADD - it adds two vectors

OUTPUT - this subroutine is used to control the
printed output.

Fig. 6. Illustration of Subroutine and Data Set Usage for Program MIKU



Program MIKU Input Instructions

Card 1

NPROB = The number of separated unrelated problems
to be run as stacked cases.

Card 2

TITLE = Title for problem (columns 1 through 80
allowed).

Card 3

NC = Number of chains

CLC = $\begin{cases} 0 & \text{First NC library chains to be used} \\ 1 & \text{NC selected library chains are to be used} \end{cases}$

METHOD = $\begin{cases} 0 & \text{Matrix exponential method is used to solve} \\ & \text{the decay equations,} \\ 1 & \text{GFAC method is used to solve the decay} \\ & \text{equations.} \end{cases}$

NTI = The number of separate time intervals for
the problem.

VOL = The volume of the Fission Product or Neutronic
System

FXSEC(1) = U-235 thermal fission cross section in barns

FXSEC(2) = U-235 fast fission cross section in barns

FXSEC(3) = U-235 high energy fission cross section in
barns

FXSEC(4) = U-238 thermal fission cross section in barns

FXSEC(5) = U-238 fast fission cross section in barns

FXSEC(6) = U-238 high energy fission cross section in
barns

FXSEC(7) = Pu-239 thermal fission cross section in
barns

FXSEC(8) = Pu-239 fast fission cross section in barns

FXSEC(9) = Pu-239 high energy fission cross section in
barns

FXSEC(10) = Pu-241 thermal fission cross section in barns

FXSEC(11) = Pu-241 fast fission cross section in barns

FXSEC(12) = Pu-241 high energy fission cross section in
barns

FXSEC(13) = U-233 thermal fission cross section in barns

FXSEC(14) = U-233 fast fission cross section in barns

FXSEC(15) = U-233 high energy fission cross section in
barns

FXSEC(16) = Th-232 thermal fission cross section in barns

FXSEC(17) = Th-232 fast fission cross section in barns

FXSEC(18) = Th-232 high energy fission cross section in
barns

Card 4 (Required if CLC = 1)

CN(I) = NC chain ID numbers

Card 5

FRATIO(1) = Thermal Flux Ratio

FRATIO(2) = Fast Flux Ratio

FRATIO(3) = High Energy Flux Ratio (10.0 to 20.0 MeV)

The above flux ratio values can be entered as ratios or as the actual flux values. For a decay interval with no buildup these ratio should be specified as zero. If the actual flux values used and P is specified as 1.0 then the Power is computed from the fission rate. If P is specified as 0.0 then the Flux ratios are set to zero. Otherwise the FR is computed from the specified reactor power.

ARATIO(1) = U-235 atom ratio

ARATIO(2) = U-238 atom ratio

ARATIO(3) = Pu-239 atom ratio

ARATIO(4) = Pu-241 atom ratio

ARATIO(5) = U-233 atom ratio

ARATIO(6) = Th-232 atom ratio

P = Reactor Power in Megawatts thermal

T = Length of the time interval in hours

DEBUG = 0 - Standard output is printed

1 Special debug information is printed

2 GFAC factors are printed

3 Triple precision results are printed

4 Special TSO output is requested

Repeat CARD 5 for each of the NTI time intervals
Repeat CARDS 2, 3, 4, and 5 as necessary for each
additional problem.

CHAPTER FIVE

Results and Discussion

The MIKU program was run for the following cases:

- a) Detailed study of Cs-137 and I-131 activities for 1000, 2000, and 3000 MW reactor power as function of reactor operation time and for different fuel enrichments (3% and 90%).

Results were obtained for both Mexp and G-factor methods.

- b) Total activity of all fission products for 1000 MW reactor power with 3% enriched fuel as function of reactor operation time using Mexp method.

- c) Comparison of CPU time for Mexp and G-factor methods for 1000 MW reactor power with 3% enriched fuel as function of reactor operation time.

- d) Total activity of all fission products as a function of time when 1000 MW reactor with 3% enriched fuel is periodically shut down and operated again.

It should be noted that when MIKU program is run with stacked cards (for different time intervals) the activity is given for the total time. For example, if three cards

are stacked together with one year time on each for some reactor power, the first result for the activity will be after one year, the second value of activity will be after two years and third value of activity will be after three years. Only in case (a) where activities of Cs-137 and I-131 are studied, the program was run with stacked cards and the time shown in the column is time punched on individual cards. Therefore the previous time intervals should be added to get correct value for time. The activity shown for one minute is therefore activity after one minute and thirty-one seconds. In cases (b) and (c) the program was run separately for different time intervals and therefore the tables for them show time without any ambiguity. In case (d) the program was run with stacked cards because of the nature of the problem.

The data used in MIKU program is given in Appendices A and B. The half lives, branching factors and fission yields for 192 chains having 798 isotopes were collected and stored in the computer.

All the results are presented in tabular form.

Table 5.1(a). Growth and Decay of Cs-137

Reactor Power 1000 MW

Fuel enrichment 3%

Mexp Method

Time	Buildup (atoms)	Decay (atoms)	Activity (Curies)
1 sec	4.192Q+16	0.0	8.249E-04
30 sec	2.706Q+18	1.484Q+17	5.617E-02
1 min	8.982Q+18	1.074Q+19	3.881E-01
30 min	2.885Q+21	1.769Q+20	5.966E+01
1 hr	6.375Q+21	3.7Q+21	1.983E+02
24 hr	1.684Q+23	1.075Q+22	3.525E+03
1 month	5.067Q+24	1.795Q+23	1.032E+05
1 yr	6.101Q+25	5.128Q+24	1.301E+06
10 yr	5.515Q+26	5.257Q+25	1.189E+07
1000 yr	2.688Q+27	6.455Q+16	5.289E+07
10 ⁵ yr	2.688Q+27	1.553Q-06	5.289E+07

Table 5.1(b)

G-Factor Method

Time	Buildup (atoms)	Decay (atoms)	Activity (Curies)
1 sec	4.192Q+16	0.0	8.249E-04
30 sec	2.706Q+18	1.484Q+17	5.617E-02
1 min	8.982Q+18	1.074Q+19	3.881E-01
30 min	2.855Q+21	1.769Q+20	5.966E+01
1 hr	6.375Q+21	3.70Q+21	1.983E+02
24 hr	1.684Q+23	1.075Q+22	3.525E+03
1 month	5.067Q+24	1.795Q+23	1.032E+05
1 yr	6.101Q+25	5.128Q+24	1.301E+06
10 yr	5.515Q+26	5.257Q+25	1.189E+07
1000 yr	2.688Q+27	6.455Q+16	5.289E+07
10 ⁵ yr	2.688Q+27	0.0	5.289E+07

Table 5.2(a)

Reactor Power 2000 MW

Fuel enrichment 3%

Mexp Method

Time	Buildup (atoms)	Decay (atoms)	Activity (Curies)
1 sec	8.385Q+16	0.0	1.650E-03
30 sec	5.413Q+18	2.968Q+17	1.123E-01
1 min	1.796Q+19	2.148Q+19	7.762E-01
30 min	5.711Q+21	3.538Q+20	1.193E+02
1 hr	1.275Q+22	7.401Q+21	3.965E+02
24 hr	3.368Q+23	2.149Q+22	7.051E+03
1 month	1.013Q+25	3.590Q+23	2.065E+05
1 yr	1.220Q+26	1.026Q+25	2.603E+06
10 yr	1.103Q+27	1.051Q+26	2.377E+07
1000 yr	5.376Q+27	1.291Q+17	1.058E+08
10 ⁵ yr	5.376Q+27	3.106Q-06	1.058E+08

Table 5.2(b)

G-Factor Method

Time	Buildup (atoms)	Decay (atoms)	Activity (Curies)
1 sec	8.358Q+16	0.0	1.650E-03
30 sec	5.413Q+18	2.968Q+17	1.123E-01
1 min	1.796Q+19	2.148Q+19	7.762E-01
30 min	5.711Q+21	3.538Q+20	1.193E+02
1 hr	1.275Q+22	7.401Q+21	3.965Q+21
24 hr	3.368Q+23	2.149Q+22	7.051E+03
1 month	1.013Q+25	3.590Q+23	2.065E+05
1 yr	1.220Q+26	1.026Q+25	2.603E+06
10 yr	1.103Q+27	1.051Q+26	2.377E+07
1000 yr	5.376Q+27	1.291Q+17	1.058E+08
10 ⁵ yr	5.376Q+27	0.0	1.058E+08

Table 5.3(a)

Reactor Power 3000 MW

Fuel enrichment 3%

Mexp Method

Time	Buildup (atoms)	Decay (atoms)	Activity (Curies)
1 sec	1.258Q+17	0.0	2.475E-03
30 sec	8.199Q+18	4.452Q+17	1.685E-01
1 min	2.695Q+19	3.222Q+19	1.164E+00
30 min	8.566Q+21	5.307Q+20	1.79E+02
1 hr	1.913Q+22	1.110Q+22	5.948E+02
24 hr	5.052Q+23	3.224Q+22	1.058E+04
1 month	1.520Q+25	5.385Q+23	3.097E+05
1 yr	1.830Q+26	1.539Q+25	3.904E+06
10 yr	1.654Q+27	1.577Q+26	3.566E+07
1000 yr	8.064Q+27	1.936Q+17	1.587E+08
10 ⁵ yr	8.064Q+27	4.659Q-06	1.587E+08

Table 5.3(b)

G-Factor Method

Time	Buildup (atoms)	Decay (atoms)	Activity (Curies)
1 sec	1.258Q+17	0.0	2.475E-03
30 sec	8.119Q+18	4.452Q+17	1.685E-01
1 min	2.695Q+19	3.222Q+19	1.164E+00
30 min	8.566Q+21	5.307Q+20	1.79E+02
1 hr	1.913Q+22	1.110Q+22	5.948E+02
24 hr	5.052Q+23	3.224Q+22	1.058E+04
1 month	1.520Q+25	5.385Q+23	3.097E+05
1 yr	1.830Q+26	1.539Q+25	3.904E+06
10 yr	1.654Q+27	1.577Q+26	3.566E+07
1000 yr	8.064Q+27	1.936Q+17	1.587E+08
10 ⁵ yr	8.064Q+27	0.0	1.587E+08

Table 5.4(a). Growth and Decay of I-131

Reactor Power 1000 MW

Fuel enrichment 3%

Mexp Method

Time	Buildup (atoms)	Decay (atoms)	Activity (Curies)
1 sec	1.288Q+15	0.0	3.474E-02
30 sec	4.771Q+16	1.92Q+15	1.338E+00
1 min	1.152Q+17	9.495Q+16	5.668E+00
30 min	1.307Q+20	1.677Q+19	3.978E+03
1 hr	6.680Q+20	9.506Q+20	4.365E+04
24 hr	6.336Q+22	4.160Q+21	1.821E+06
1 month	8.193Q+23	6.014Q+21	2.226E+07
1 yr	8.879Q+23	1.831Q+10	2.395E+07
10 yr	8.879Q+23	0.0	2.395E+07
1000 yr	8.879Q+23	0.0	2.395E+07
10 ⁵ yr	8.879Q+23	0.0	2.395E+07

Table 5.4(b)

G-Factor Method

Time	Buildup (atoms)	Decay (atoms)	Activity (Curies)
1 sec	1.288Q+15	0.0	3.474E-02
30 sec	4.771Q+16	1.920Q+15	1.338E+00
1 min	1.152Q+17	9.495Q+16	5.668E+00
30 min	1.307Q+20	1.677Q+19	3.978E+03
1 hr	6.680Q+20	9.506Q+20	4.365E+04
24 hr	6.336Q+22	4.160Q+21	1.821E+06
1 month	8.193Q+23	6.014Q+21	2.226E+07
1 yr	8.879Q+23	1.831Q+10	2.395E+07
10 yr	8.879Q+23	0.0	2.395E+07
1000 yr	8.879Q+23	0.0	2.395E+07
10 ⁵ yr	8.879Q+23	0.0	2.395E+07

Table 5.5(a)

Reactor Power 2000 MW

Fuel enrichment 3%

Mexp Method

Time	Buildup (atoms)	Decay (atoms)	Activity (Curies)
1 sec	2.576Q+15	0.0	6.947E-02
30 sec	9.542Q+16	3.841Q+15	2.677E+00
1 min	2.304Q+17	1.899Q+17	1.134E+01
30 min	2.615Q+20	3.354Q+19	7.955E+03
1 hr	1.336Q+21	1.901Q+21	8.730E+04
24 hr	1.267Q+23	8.32Q+21	3.642E+06
1 month	1.639Q+24	1.203Q+22	4.451E+07
1 yr	1.776Q+24	3.662Q+10	4.789E+07
10 yr	1.776Q+24	0.0	4.789E+07
1000 yr	1.776Q+24	0.0	4.789E+07
10 ⁵ yr	1.776Q+24	0.0	4.789E+07

Table 5.5(b)

G-Factor Method

Time	Buildup (atoms)	Decay (atoms)	Activity (Curies)
1 sec	2.576Q+15	0.0	6.947E-02
30 sec	9.542Q+16	3.841Q+15	2.677E+00
1 min	2.304Q+17	1.899Q+17	1.134E+01
30 min	2.615Q+20	3.354Q+19	7.955E+03
1 hr	1.336Q+21	1.901Q+21	8.730E+04
24 hr	1.267Q+23	8.320Q+21	3.642E+06
1 month	1.639Q+24	1.203Q+22	4.451E+07
1 yr	1.776Q+24	3.662Q+10	4.789E+07
10 yr	1.776Q+24	0.0	4.789E+07
1000 yr	1.776Q+24	0.0	4.789E+07
10 ⁵ yr	1.776Q+24	0.0	4.789E+07

Table 5.6(a)

Reactor Power 3000 MW

Fuel enrichment 3%

Mexp Method

Time	Buildup (atoms)	Decay (atoms)	Activity (Curies)
1 sec	3.864Q+15	0.0	1.042E-01
30 sec	1.431Q+17	5.761Q+15	4.015E+00
1 min	3.457Q+17	2.849Q+17	1.70E+01
30 min	3.922Q+20	5.030Q+19	1.193E+04
1 hr	2.004Q+21	2.852Q+21	1.310E+05
24 hr	1.901Q+23	1.248Q+22	5.463E+06
1 month	2.458Q+24	1.804Q+22	6.677E+07
1 yr	2.664Q+24	5.493Q+10	7.184E+07
10 yr	2.664Q+24	0.0	7.184E+07
1000 yr	2.664Q+24	0.0	7.184E+07
10 ⁵ yr	2.664Q+24	0.0	7.184E+07

Table 5.6(b)

G-Factor Method

Time	Buildup (atoms)	Decay (atoms)	Activity (Curies)
1 sec	3.864Q+15	0.0	1.042E-01
30 sec	1.431Q+17	5.761Q+15	4.015E+00
1 min	3.457Q+17	2.849Q+17	1.7E+01
30 min	3.922Q+20	5.030Q+19	1.193E+04
1 hr	2.004Q+21	2.852Q+21	1.310E+05
24 hr	1.901Q+23	1.248Q+22	5.463E+06
1 month	2.458Q+24	1.804Q+22	6.677E+07
1 yr	2.664Q+24	5.493Q+10	7.184E+07
10 yr	2.664Q+24	0.0	7.184E+07
1000 yr	2.664Q+24	0.0	7.184E+07
10 ⁵ yr	2.664Q+24	0.0	7.184E+07

Table 5.7(a). Growth and Decay of Cs-137

Reactor Power 1000 MW

Fuel enrichment 90%

Mexp Method

Time	Buildup (atoms)	Decay (atoms)	Activity (Curies)
1 sec	5.401Q+16	0.0	1.063E-03
30 sec	3.178Q+18	1.665Q+17	6.581E-02
1 min	1.00Q+19	1.127Q+19	4.187E-01
30 min	2.825Q+21	1.744Q+20	5.901E+01
1 hr	6.295Q+21	3.648Q+21	1.957E+02
24 hr	1.661Q+23	1.059Q+22	3.476E+03
1 month	4.996Q+24	1.770Q+23	1.018E+05
1 yr	6.016Q+25	5.056Q+24	1.283E+06
10 yr	5.437Q+26	5.184Q+25	1.172E+07
1000 yr	2.650Q+27	6.364Q+16	5.215E+07
10 ⁵ yr	2.650Q+27	1.531Q-06	5.215E+07

Table 5.7(b)

G-Factor Method

Time	Buildup (atoms)	Decay (atoms)	Activity (Curies)
1 sec	5.401Q+16	0.0	1.063E-03
30 sec	3.178Q+18	1.665Q+17	6.581E-02
1 min	1.00Q+19	1.127Q+19	4.187E-01
30 min	2.825Q+21	1.744Q+20	5.901E+01
1 hr	6.295Q+21	3.648Q+21	1.957E+02
24 hr	1.661Q+23	1.059Q+22	3.476E+03
1 month	4.996Q+24	1.770Q+23	1.018E+05
1 yr	6.016Q+25	5.056Q+24	1.283E+06
10 yr	5.437Q+26	5.184Q+25	1.172E+07
1000 yr	2.650Q+27	6.364Q+16	5.215E+07
10 ⁵ yr	2.650Q+27	0.0	5.215E+07

Table 5.8(a)

Reactor Power 2000 MW

Fuel enrichment 90%

Mexp Method

Time	Buildup (atoms)	Decay (atoms)	Activity (Curies)
1 sec	1.080Q+17	0.0	2.126E-03
30 sec	6.356Q+18	3.329Q+17	1.316E-01
1 min	2.001Q+19	2.255Q+19	8.374E-01
30 min	5.649Q+21	3.489Q+20	1.180E+02
1 hr	1.259Q+22	7.297Q+21	3.913E+02
24 hr	3.321Q+23	2.119Q+22	6.952E+03
1 month	9.992Q+24	3.539Q+23	2.036E+05
1 yr	1.203Q+26	1.011Q+25	2.566E+06
10 yr	1.087Q+27	1.037Q+26	2.344E+07
1000 yr	5.301Q+27	1.273Q+17	1.043E+08
10 ⁵ yr	5.301Q+27	3.063Q-06	1.043E+08

Table 5.8(b)

G-Factor Method

Time	Buildup (atoms)	Decay (atoms)	Activity (Curies)
1 sec	1.080Q+17	0.0	2.126E-03
30 sec	6.356Q+18	3.329Q+17	1.316E-01
1 min	2.001Q+19	2.255Q+19	8.374E-01
30 min	5.649Q+21	3.489Q+20	1.180E+02
1 hr	1.259Q+22	7.297Q+21	3.913E+02
24 hr	3.321Q+23	2.119Q+22	6.952E+03
1 month	9.992Q+24	3.539Q+23	2.036E+05
1 yr	1.203Q+26	1.011Q+25	2.566E+06
10 yr	1.087Q+27	1.037Q+26	2.344E+07
1000 yr	5.301Q+27	1.273Q+17	1.043E+08
10 ⁵ yr	5.301Q+27	0.0	1.043E+08

Table 5.9(a)

Reactor Power 3000 MW

Fuel enrichment 90%

Mexp Method

Time	Buildup (atoms)	Decay (atoms)	Activity (Curies)
1 sec	1.620Q+17	0.0	3.188E-03
30 sec	9.534Q+18	4.994Q+17	1.974E-03
1 min	3.001Q+19	3.382Q+19	1.256E+00
30 min	8.474Q+21	5.233Q+20	1.770E+02
1 hr	1.889Q+22	1.095Q+22	5.870E+02
24 hr	4.982Q+23	3.178Q+22	1.043E+04
1 month	1.499Q+25	5.309Q+23	3.054E+05
1 yr	1.805Q+26	1.517Q+25	3.85E+06
10 yr	1.631Q+27	1.555Q+26	3.516E+07
1000 yr	7.951Q+27	1.909Q+17	1.565E+08
10 ⁵ yr	7.951Q+27	4.594Q-06	1.565E+08

Table 5.9(b)

G-Factor Method

Time	Buildup (atoms)	Decay (atoms)	Activity (Curies)
1 sec	1.620Q+17	0.0	3.188E-03
30 sec	9.534Q+18	4.994Q+17	1.974E-03
1 min	3.001Q+19	3.382Q+19	1.256E+00
30 min	8.474Q+21	5.233Q+20	1.770E+02
1 hr	1.889Q+22	1.095Q+22	5.870E+02
24 hr	4.982Q+23	3.178Q+22	1.043E+04
1 month	1.499Q+25	5.309Q+23	3.054E+05
1 yr	1.805Q+26	1.517Q+25	3.85E+06
10 yr	1.631Q+27	1.555Q+26	3.516E+07
1000 yr	7.951Q+27	1.909Q+17	1.565E+08
10 ⁵ yr	7.951Q+27	0.0	1.565E+08

Table 5.10(a). Growth and Decay of I-131
 Reactor Power 1000 MW Fuel enrichment 90%

Mexp Method

Time	Buildup (atoms)	Decay (atoms)	Activity (Curies)
1 sec	1.470Q+15	0.0	3.964E-02
30 sec	6.014Q+16	2.581Q+15	1.692E+00
1 min	1.543Q+17	1.380Q+17	7.882E+00
30 min	1.606Q+20	1.991Q+19	4.868E+03
1 hr	7.861Q+20	1.088Q+21	5.055E+04
24 hr	7.191Q+22	4.728Q+21	2.067E+06
1 month	9.366Q+23	6.888Q+21	2.544E+07
1 yr	1.015Q+24	2.097Q+10	2.738E+07
10 yr	1.015Q+24	0.0	2.738E+07
1000 yr	1.015Q+24	0.0	2.738E+07
10 ⁵ yr	1.015Q+24	0.0	2.738E+07

Table 5.10(b)

G-Factor Method

Time	Buildup (atoms)	Decay (atoms)	Activity (Curies)
1 sec	1.470Q+15	0.0	3.964E-02
30 sec	6.014Q+16	2.581Q+15	1.692E+00
1 min	1.543Q+17	1.380Q+17	7.882E+00
30 min	1.606Q+20	1.991Q+19	4.868E+03
1 hr	7.861Q+20	1.088Q+21	5.055E+04
24 hr	7.191Q+22	4.728Q+21	2.067E+06
1 month	9.366Q+23	6.888Q+21	2.544E+07
1 yr	1.015Q+24	2.097Q+10	2.738E+07
10 yr	1.015Q+24	0.0	2.738E+07
1000 yr	1.015Q+24	0.0	2.738E+07
10 ⁵ yr	1.015Q+24	0.0	2.738E+07

Table 5.11(a)

Reactor Power 2000 MW

Fuel enrichment 90%

Mexp Method

Time	Buildup (atoms)	Decay (atoms)	Activity (Curies)
1 sec	2.940Q+15	0.0	7.929E-02
30 sec	1.203Q+17	5.163Q+15	3.383E+00
1 min	3.086Q+17	2.759Q+17	1.576E+01
30 min	3.212Q+20	3.982Q+19	9.736E+03
1 hr	1.572Q+21	2.177Q+21	1.011E+05
24 hr	1.438Q+23	9.457Q+21	4.134E+06
1 month	1.873Q+24	1.378Q+22	5.089E+07
1 yr	2.030Q+24	4.194Q+10	5.476E+07
10 yr	2.030Q+24	0.0	5.476E+07
1000 yr	2.030Q+24	0.0	5.476E+07
10 ⁵ yr	2.030Q+24	0.0	5.476E+07

Table 5.11(b)

G-Factor Method

Time	Buildup (atoms)	Decay (atoms)	Activity (Curies)
1 sec	2.940Q+15	0.0	7.929E-02
30 sec	1.203Q+17	5.163Q+15	3.383E+00
1 min	3.086Q+17	2.759Q+17	1.576E+01
30 min	3.212Q+20	3.982Q+19	9.736E+03
1 hr	1.572Q+21	2.177Q+21	1.011E+05
24 hr	1.438Q+23	9.457Q+21	4.134E+06
1 month	1.873Q+24	1.378Q+22	5.089E+07
1 yr	2.030Q+24	4.194Q+10	5.476E+07
10 yr	2.030Q+24	0.0	5.476E+07
1000 yr	2.030Q+24	0.0	5.476E+07
10 ⁵ yr	2.030Q+24	0.0	5.476E+07

Table 5.12(a)

Reactor Power 3000 MW

Fuel enrichment 90%

Mexp Method

Time	Buildup (atoms)	Decay (atoms)	Activity (Curies)
1 sec	4.410Q+15	0.0	1.189E-01
30 sec	1.804Q+17	7.744Q+15	5.075E+00
1 min	4.629Q+17	4.139Q+17	2.365E+01
30 min	4.818Q+20	5.973Q+19	1.460E+04
1 hr	2.358Q+21	3.265Q+21	1.517E+05
24 hr	2.157Q+23	1.419Q+22	6.200E+06
1 month	2.810Q+24	2.066Q+22	7.633E+07
1 yr	3.046Q+24	6.291Q+10	8.214E+07
10 yr	3.046Q+24	0.0	8.214E+07
1000 yr	3.046Q+24	0.0	8.214E+07
10 ⁵ yr	3.046Q+24	0.0	8.214E+07

Table 5.12(b)

G-Factor Method

Time	Buildup (atoms)	Decay (atoms)	Activity (Curies)
1 sec	4.410Q+15	0.0	1.189E-01
30 sec	1.804Q+17	7.744Q+15	5.075E+00
1 min	4.629Q+17	4.139Q+17	2.365E+01
30 min	4.818Q+20	5.973Q+19	1.460E+04
1 hr	2.358Q+21	3.265Q+21	1.517E+05
24 hr	2.157Q+23	1.419Q+22	6.200E+06
1 month	2.810Q+24	2.066Q+22	7.633E+07
1 yr	3.046Q+24	6.291Q+10	8.214E+07
10 yr	3.046Q+24	0.0	8.214E+07
1000 yr	3.046Q+24	0.0	8.214E+07
10 ⁵ yr	3.046Q+24	0.0	8.214E+07

Table 5.13
Study of Cs-137
Fuel Enrichment 90%

Time (Years)	<u>Reactor Power 1000 MW</u>	<u>Reactor Power 3000 MW</u>
	Activity (Curies)	Activity (Curies)
4	4.576E+06	1.373E+07
10	1.070E+07	3.21E+07
20	1.92E+07	5.761E+07
30	2.596E+07	7.88E+07
40	3.133E+07	9.4E+07
50	3.560E+07	1.068E+08
60	3.90E+07	1.170E+08
70	4.170E+07	1.251E+08
80	4.384E+07	1.315E+08
90	4.555E+07	1.366E+08
100	4.690E+07	1.407+08
110	4.798E+07	1.439E+08
120	4.883E+07	1.465E+08
130	4.951E+07	1.485E+08
140	5.006E+07	1.502E+08
150	5.049E+07	1.515E+08

Table 5.14
Study of I-131
Fuel Enrichment 90%

Time (Days)	<u>Reactor Power 1000 MW</u>	<u>Reactor Power 3000 MW</u>
	Activity (Curies)	Activity (Curies)
2	3.906E+06	1.172E+07
4	7.508E+06	2.252E+07
6	1.062E+07	3.158E+07
8	1.326E+07	3.978E+07
10	1.549E+07	4.647E+07
12	1.737E+07	5.212E+07
14	1.896E+07	5.687E+07
16	2.029E+07	6.087E+07
18	2.141E+07	6.424E+07
20	2.236E+07	6.707E+07
22	2.315E+07	6.946E+07
24	2.382E+07	7.147E+07
26	2.439E+07	7.316E+07
28	2.486E+07	7.458E+07
30	2.526E+07	7.578E+07

Table 5.15

Study of Total Activity

Reactor Power 1000 MW Fuel Enrichment 3%

Time (hours)	Activity of all Fission Products (Curies)
2.77E-04	1.416E+08
1.0E+01	1.145E+09
2.0E+01	1.204E+09
4.0E+01	1.257E+09
8.0E+01	1.300E+09
2.0E+02	1.352E+09
4.0E+02	1.390E+09

Table 5.16

Comparison of CPU Time Taken by G-Factor and Mexp Methods
 (Reactor Power 1000 Mw, Fuel Enrichment 3%)
 for Total Activity of all Fission Products

Reactor Operation time (hours)	Mexp Method CPU time (min)	G-Factor Method CPU time (min)
2.77E-04	0.29	0.28
8.333E-03	0.30	0.28
0.0166	0.30	0.28
0.5	0.33	0.28
1.0	0.33	0.28
24.0	0.35	0.28
720.0	0.38	0.28
8760.0	0.40	0.28
8.76E+04	0.42	0.28
8.76E+06	0.46	0.28
8.76E+08	0.50	0.28

Fig. 7. Activity of Cs-137 for 1000 and 3000 MW reactor power (90% fuel enrichment) vs. time.

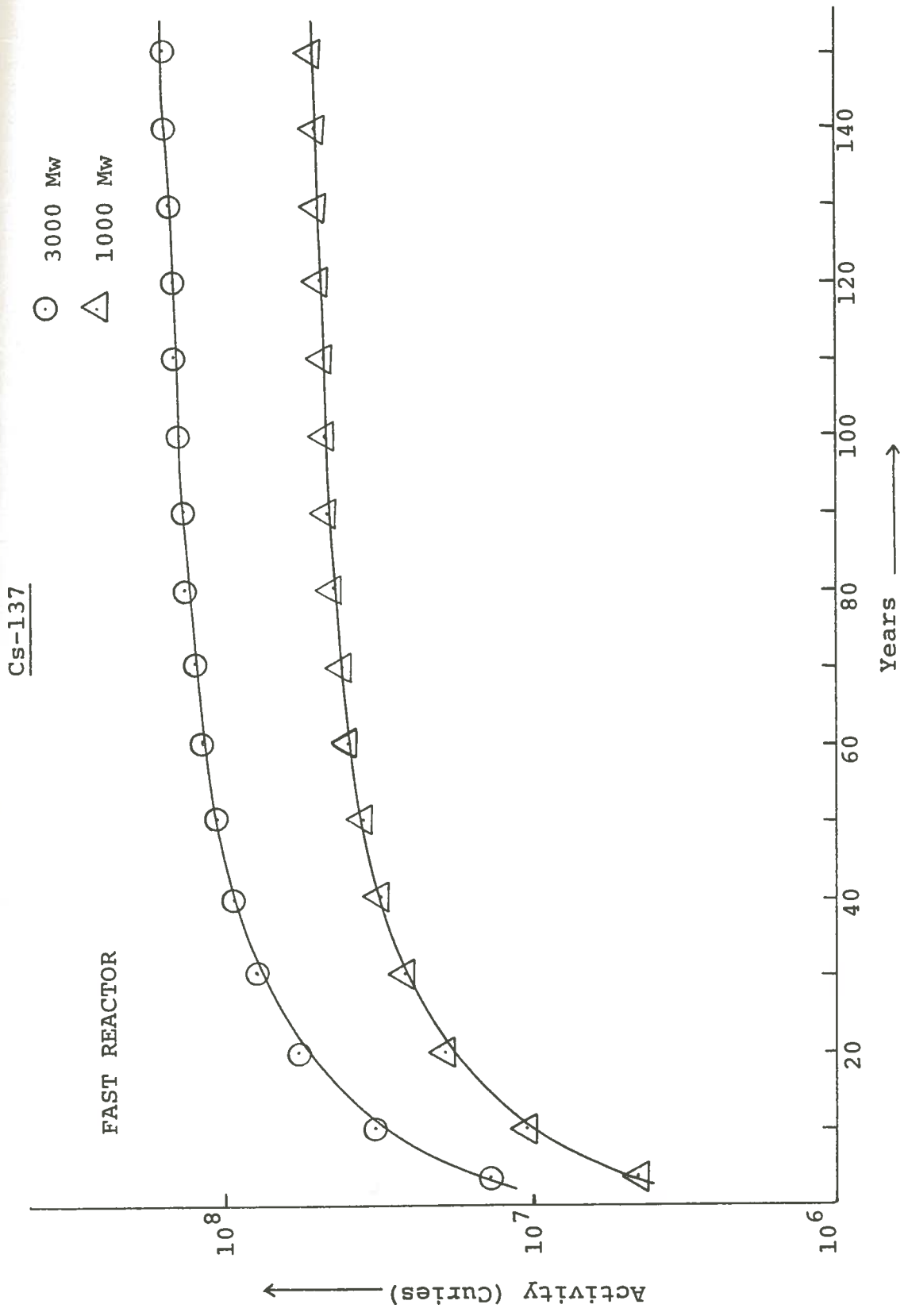


Fig. 8. Activity of I-131 for 1000 and 3000 MW reactor power (90% fuel enrichment) vs. time.

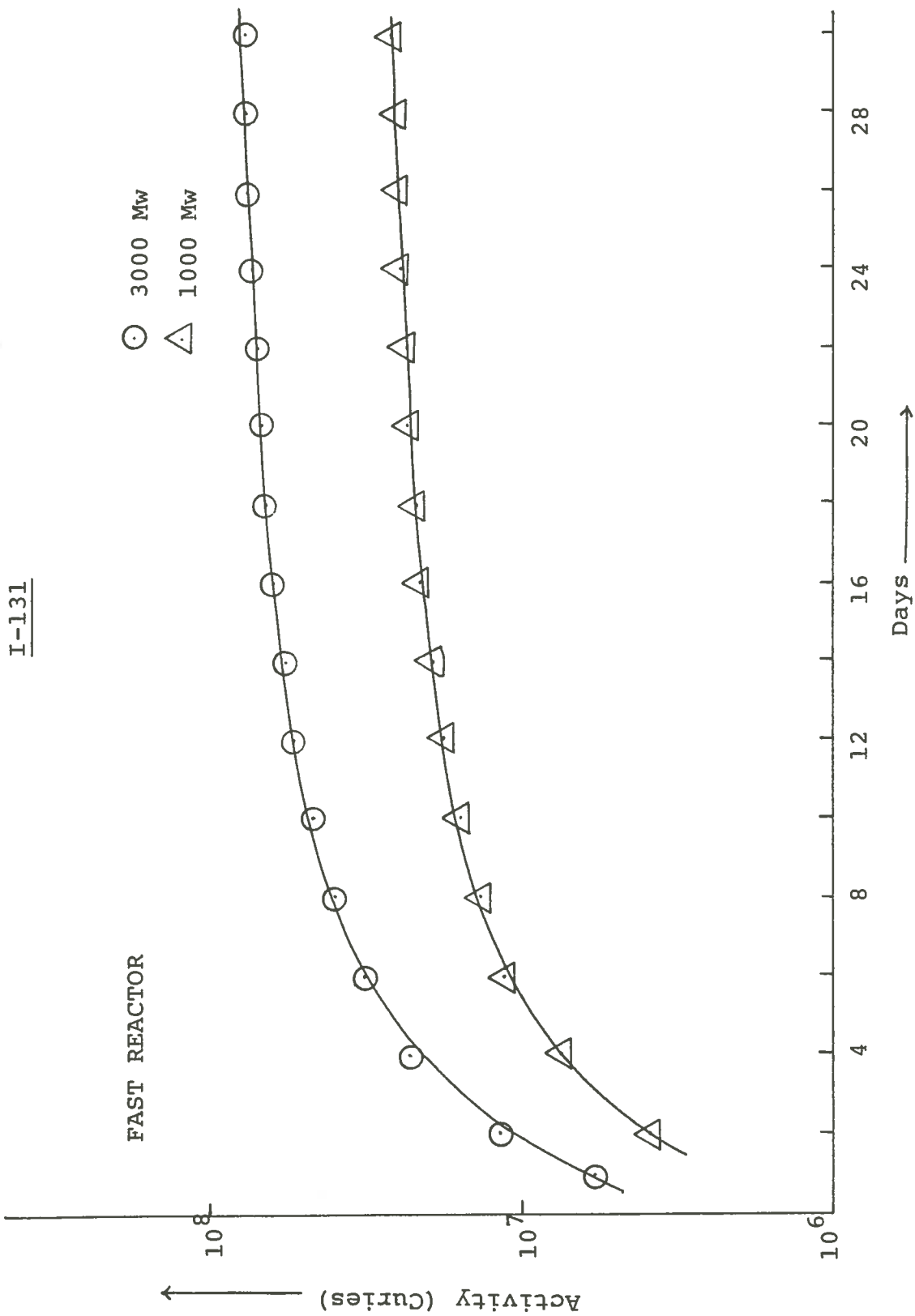
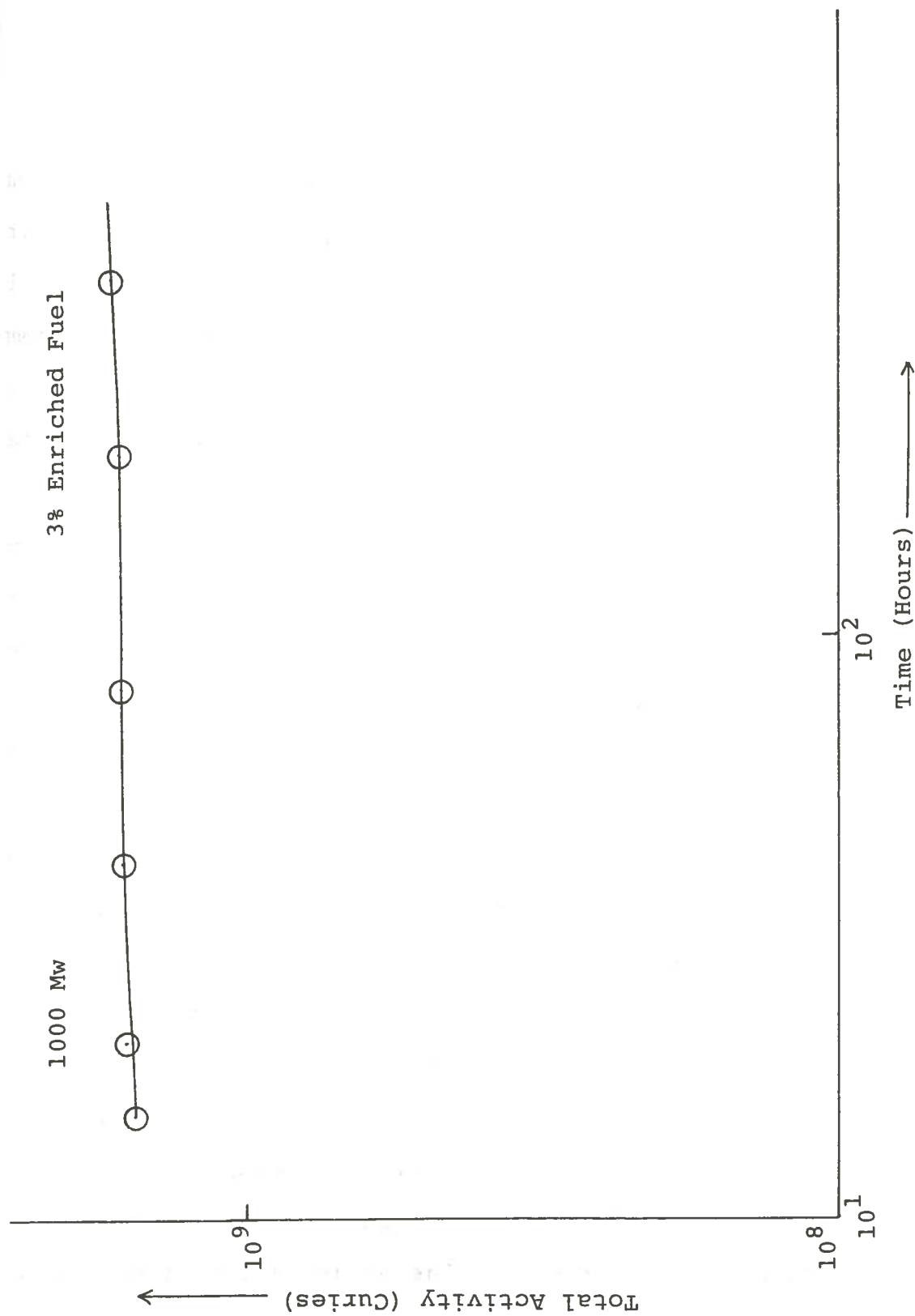


Fig. 9. Total activity of all fission products (1000 MW reactor power, 3% fuel enrichment) vs. time.



DISCUSSION

It is seen that the matrix exponential and G-factor methods give the same inventory for the fission products indicating thereby that the approach to a problem is not important as long as it is based on sound reasoning. Both methods give exactly the same result upto four significant figures listed. Typically, the result agree to 10 significant figures or more.

The first step in the project was to study the matrix exponential and G-factor methods and to get a good understanding of the techniques involved. Both methods have strong and weak points. The calculations for the matrix exponential method are much more involved and are based on the solution of matrix differential equations. On the other hand, the concepts used in the G-factor method are extremely simple. Moreover, the G-factor method uses less CPU time as compared to that for matrix exponential method. The G-factor method, however, suffers from the limitation that it is unable to handle reverse branching and, therefore, does not provide a general solution to the decay problem.

The data (branching factors, half lives and fission yields) for 192 decay chains involving 798 isotopes were collected from literature and are reproduced in Appendices A and B. A stable isotope was assumed to have the half-

life of 10^{15} years. This data was stored in the computer and was used in computations based on the program MIKU which was developed by us for our specific use.

The program MIKU was run for different operating conditions of the reactor and the results are listed in Table 5.1(a) to 5.16). Table 5.1(a) to Table 5.14 give the activities of Cs-137 and I-131 for different fuel concentrations and reactor power for various reactor operating times. The results for both the methods have been listed. It should be noted that I-131 is getting contributions from two members of decay chain (Appendix A) namely, M_{Te} and Te and, therefore, activity of I-131 is obtained without neglecting any contribution. Cs-137 however, gets contribution only from Xe-137. MIKU is a very powerful and accurate computer program as it includes all branches (contributions).

Tables 5.1(a) to 5.14 show that when the reactor power is increased by a factor of two or three, the activity is also increased by the same factor. This result was expected because of the linearity of the equation used to calculate the activity of the fission products.

Activities of Cs-137 and I-131 (Table 5.13 and 5.14) are, respectively, plotted in Fig. 7 and 8. It is observed that the activity of Cs-137 saturates after about 100 years of reactor operation whereas I-131 gets saturated in about 20 days. This occurrence of saturation is

supported by theory (Ref. 6). It can be shown that when the rate of production of the parent atom is constant in time, the activity of daughter atom reaches saturation after 3 or 4 times its half-life. Cs-137 and I-131 have their respective half-lives as 30.17 years and 8.04 days and hence their saturation times agree with the theoretical predictions.

Total activity for all the isotopes as function of time for 1000 MW reactor power and 3% enriched fuel have been computed and are listed in Table 5.15. The results are also displayed graphically (Fig. 9).

One can derive important information which can be very useful in radioactive waste disposal problem. One can easily obtain the remaining activity in a radioactive waste after certain length of time by simply putting the reactor operating power as zero and follow the decay chains for that length of time. This situation corresponds to the natural radioactive decay process. One can, therefore, easily estimate the time for which the radioactive waste remains unsafe.

The CPU time required by both the methods are also compared (Table 5.16). It is seen that the G-factor method takes 0.28 minutes CPU time to calculate the total activity of all the 798 isotopes for reactor operating time from one second to 10^5 years. The Mexp method,

however, requires different CPU time depending on the reactor power. Comparison of CPU time show that the G-factor method is better computationally.

A case study was done for a reactor which is operated for one year (1000 MW power, 3% fuel enrichment) and then shut down for one year (same power and fuel concentration). The program was run with three stacked cards for three one-year time intervals. The total CPU time for G-factor method was 0.37 minutes and for Mexp method was 0.74 minutes. This shows that when the time intervals are equal the computer does not take three times the time for one interval but considerably less. This is because some factors, which are common for a particular interval, are not calculated in each computation, but only in the first.

Further study can be done by incorporating subroutine GSPEC which will compute gamma spectrum and can help in determining source term for shielding calculations. Activation libraries can also be included. Fuel management and fuel depletion routines can be the next further step in the study. To make the program more general, actinide chain should also be included.

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APPENDIX A

Fission Product Chains

Isotope	Chain	ID	Half Life	BF(1)	BF(2)	BF(3)	Decay Chain
Ni-66	1	1	54.8 H	1.0	0.0	0.0	Ni $\xrightarrow{54.8h}$ Cu $\xrightarrow{5.10m}$ Zn
Cu-66	1	2	5.10 M	1.0	0.0	0.0	
Zn-66	1	3	10 ¹⁵ Y	1.0	0.0	0.0	
Ni-67	2	4	18.0S	1.0	0.0	0.0	Ni $\xrightarrow{18S}$ Cu $\xrightarrow{61.9h}$ Zn
Cu-67	2	5	61.9 H	1.0	0.0	0.0	
Zn-67	2	6	10 ¹⁵ Y	1.0	0.0	0.0	
Cu-68M	3	7	3.8 M	0.5	0.5	0.0	
Cu-68	3	8	31 S	1.0	0.0	0.0	M _{Cu} $\xrightarrow{3.8M}$ Cu $\xrightarrow{31S}$ Zn
Zn-68	3	9	10 ¹⁵ Y	1.0	0.0	0.0	
Ga-68	4	10	68.1 M	1.0	0.0	0.0	
Zn-68	4	11	10 ¹⁵ Y	1.0	0.0	0.0	Ga $\xrightarrow{68.1M}$ Zn
Cu-69	5	12	3.0 M	0.0	1.0	0.0	
Zn-69M	5	13	14.0 H	1.0	0.0	0.0	
Zn-69	5	14	56.0 M	1.0	0.0	0.0	
Ga-69	5	15	10 ¹⁵ Y	1.0	0.0	0.0	Cu $\xrightarrow{3.0M}$ Zn $\xrightarrow{14.0H}$ Zn $\xrightarrow{56M}$ Ga
Cu-70M	6	16	47.0 s	0.0	1.0	0.0	
Cu-70	6	17	5.0 s	1.0	0.0	0.0	
Zn-70	6	18	10 ¹⁵ Y	1.0	0.0	0.0	M _{Cu} $\xrightarrow{47S}$ Cu $\xrightarrow{5S}$ Zn
Ga-70	7	19	21.1 M	1.0	0.0	0.0	
Ge-70	7	20	10 ¹⁵ Y	1.0	0.0	0.0	Ga $\xrightarrow{21.1M}$ Ge

Isotope	Chain	ID	Half Life	BF(1)	BF(2)	BF(3)	Decay Chain
Zn-71M	8	21	3.9 H	.0005	.9995	0.0	$ \begin{array}{c} \text{M}_{\text{Zn}} \xrightarrow{3.9\text{H}} \text{Zn} \xrightarrow{2.4\text{M}} \text{Ga} \\ \text{Zn} \xrightarrow{.0005} \text{Ga} \end{array} $
Zn-71	8	22	2.4 M	1.0	0.0	0.0	
Ga-71	8	23	10^{15}y	1.0	0.0	0.0	
As-71	9	24	61.0 H	1.0	0.0	0.0	$ \text{As} \xrightarrow{61\text{H}} \text{Ge} \xrightarrow{11.2\text{D}} \text{Ga} $
Ge-71	9	25	11.2 D	1.0	0.0	0.0	
Ga-71	9	26	10^{15}y	1.0	0.0	0.0	
Zn-72	10	27	45.6 H	1.0	0.0	0.0	$ \text{Zn} \xrightarrow{45.6\text{H}} \text{Ga} \xrightarrow{14.1\text{H}} \text{Ge} $
Ga-72	10	28	14.1 H	1.0	0.0	0.0	
Ge-72	10	29	10^{15}y	1.0	0.0	0.0	
As-72	11	30	26.0 H	1.0	0.0	0.0	$ \text{As} \xrightarrow{26.0\text{H}} \text{Ge} $
Ge-72	11	31	10^{15}y	1.0	0.0	0.0	
Zn-73	12	32	24.0 s	1.0	0.0	0.0	$ \text{Zn} \xrightarrow{24\text{s}} \text{Ga} \xrightarrow{4.87\text{H}} \text{Ge} $
Ga-73	12	33	4.87 H	1.0	0.0	0.0	
Ge-73	12	34	10^{15}y	1.0	0.0	0.0	
As-73	13	35	80.3 D	1.0	0.0	0.0	$ \text{As} \xrightarrow{80.3\text{D}} \text{Ge} $
Ge-73	13	36	10^{15}y	1.0	0.0	0.0	
Zn-74	14	37	95.0 s	1.0	0.0	0.0	$ \text{Zn} \xrightarrow{955} \text{Ga} \xrightarrow{105} \text{M}_{\text{Ge}} \xrightarrow{8.1\text{M}} \text{Ge} $
Ga-74	14	38	10 s	1.0	0.0	0.0	
Ge-74M	14	39	8.1 M	1.0	0.0	0.0	
Ge-74	14	40	10 y	1.0	0.0	0.0	

Isotope	Chain	ID	Half life	BF(1)	BF(2)	BF(3)	Decay Chain
As-74M	15	41	8.0 s	1.0	0.0	0.0	
As-74	15	42	17.78 D	.673	.327	0.0	
Ge-74	15	43	10 ¹⁵ y	1.0	0.0	0.0	MAs 8.05 → As 17.78D → Ge → Se
Se-74	15	44	10 ¹⁵ y	1.0	0.0	0.0	As 17.78D → Ge → Se
Zn-75	16	45	10.2 s	1.0	0.0	0.0	
Ga-75	16	46	2.1 M	.04	.96	0.0	
Ge-75M	16	47	48.0 s	1.0	0.0	0.0	
Ge-75	16	48	82.8 M	1.0	0.0	0.0	
As-75	16	49	10 ¹⁵ y	1.0	0.0	0.0	Zn 10.2s → Ga 2.1M → Ge 48S → Ge 82.8M → As
Se-75	17	50	118.5 D	1.0	0.0	0.0	
As-75	17	51	10 ¹⁵ y	1.0	0.0	0.0	Se 118.5D → As
Zn-76	18	52	5.7 s	1.0	0.0	0.0	
Ga-76	18	53	27.1 s	1.0	0.0	0.0	
Ge-76	18	54	10 ¹⁵ y	1.0	0.0	0.0	Zn 5.7s → Ga 27.1s → Ge
As-76	19	55	26.3 H	1.0	0.0	0.0	
Se-76	19	56	10 ¹⁵ y	1.0	0.0	0.0	As 26.3H → Se
Zn-77	20	57	1.4 s	1.0	0.0	0.0	
Ga-77	20	58	13.0 s	.88	.12	0.0	
Ge-77M	20	59	53.0 s	.21	.79	0.0	
Ge-77	20	60	11.3 H	1.0	0.0	0.0	
As-77	20	61	38.8 H	.003	.997	0.0	
Se-77M	20	62	17.4 s	1.0	0.0	0.0	
Se-77	20	63	10 ¹⁵ y	1.0	0.0	0.0	Zn 1.4s → Ga .13s → Ge .53s → Ge 11.3H → As 38.8H Ga .88 → Ge .21 → Se .79 MSe 17.4s → Se

Isotope	Chain	ID	Half Life	BF(1)	BF(2)	BF(3)	Decay Chain
Br-77M	21	64	4.3 M	1.0	0.0	0.0	
Br-77	21	65	57.0 H	.01	.99	0.0	
Se-77M	21	66	17.4 s	1.0	0.0	0.0	Br $\xrightarrow{4.3M}$ M $\xrightarrow{57.0H}$ Br $\xrightarrow{.01}$ Se $\xrightarrow{17.4s}$ Se
Se-77	21	67	10 ¹⁵ y	1.0	0.0	0.0	
Ga-78	22	68	5.1 s	1.0	0.0	0.0	
Ge-78	22	69	1.45 H	1.0	0.0	0.0	
As-78	22	70	10 ¹⁵ y	1.0	0.0	0.0	Ga $\xrightarrow{5.1s}$ Ge $\xrightarrow{1.45H}$ As $\xrightarrow{91M}$ Se
Se-78	22	71	10 ¹⁵ y	1.0	0.0	0.0	
Br-78	23	72	6.46 M	1.0	0.0	0.0	
Se-78	23	73	10 ¹⁵ y	1.0	0.0	0.0	Br $\xrightarrow{6.46M}$ Se
Ga-79	24	74	3.0 s	1.0	0.0	0.0	
Ge-79	24	75	42.0 s	1.0	0.0	0.0	
As-79	24	76	9.0 M	1.0	0.0	0.0	Ga $\xrightarrow{3.0s}$ Ge $\xrightarrow{42s}$ As $\xrightarrow{9.0M}$ Se $\xrightarrow{3.9M}$
Se-79M	24	77	3.9 M	1.0	0.0	0.0	
Se-79	24	78	6.5x10 ⁴ y	0.0	1.0	0.0	
Br-79M	24	79	4.9 s	1.0	0.0	0.0	Se $\xrightarrow{6.5x10^4}$ Br $\xrightarrow{4.9s}$ Br
Br-79	24	80	10 ¹⁵ y	1.0	0.0	0.0	
Kr-79M	25	81	50.0 s	1.0	0.0	0.0	
Kr-79	25	82	35.0 H	1.0	0.0	0.0	
Br-79	25	83	10 ¹⁵ y	1.0	0.0	0.0	Kr $\xrightarrow{50s}$ Kr $\xrightarrow{35H}$ Br
Ga-80	26	84	1.66 s	1.0	0.0	0.0	
Ge-80	26	85	29.0 s	1.0	0.0	0.0	
As-80	26	86	16.0 s	1.0	0.0	0.0	Ga $\xrightarrow{1.66s}$ Ge $\xrightarrow{29s}$ As $\xrightarrow{16s}$ Se
Se-80	26	87	10 ¹⁵ y	1.0	0.0	0.0	

Isotope	Chain	ID	Half Life	BF(1)	BF(2)	BF(3)	Decay Chain
Br-80M	27	88	4.42 H	1.0	0.0	0.0	
Br-80	27	89	17.6 M	.914	.086	0.0	
Kr-80	27	90	10 ¹⁵ y	1.0	0.0	0.0	Br ^M 4.42H → Br ^M 17.6M → Kr ^M .914 → Se ^M .086
Se-80	27	91	10 ¹⁵ y	1.0	0.0	0.0	
Ga-81	28	92	1.25 s	1.0	0.0	0.0	
Ge-81	28	93	10.0 s	1.0	0.0	0.0	
As-81	28	94	33.0 s	0.0	1.0	0.0	
Se-81M	28	95	57.3 M	1.0	0.0	0.0	
Se-81	28	96	18.5 M	1.0	0.0	0.0	
Br-81	28	97	10 ¹⁵ y	1.0	0.0	0.0	Ga ^M 1.2s, Ge ^M 10s → As ^M 33s, Se ^M 57.3M → Se ^M 18.5M → Br ^M 18.5M
Rb-81M	29	98	32.0 M	0.5	0.0	0.5	
Rb-81	29	99	4.58 H	.75	.25	0.0	
Kr-81M	29	100	13.0 s	1.0	0.0	0.0	
Kr-81	29	101	2.1x10 ⁵ y	1.0	0.0	0.0	
Br-81	29	102	10 ¹⁵ y	1.0	0.0	0.0	Rb ^M 32M → Rb ^M 4.58H → Kr ^M 13s → Kr ^M 2.1x10 ⁵ y → Br ^M .75
Ge-82	30	103	4.6 s	0.0	1.0	0.0	
As-82M	30	104	19.0 s	0.2	0.8	0.0	
As-82	30	105	14.0 s	1.0	0.0	0.0	
Se-82	30	106	1.4x10 ²⁰ y	1.0	0.0	0.0	Ge ^M 0.0 → As ^M .20 → As ^M 1.0 → Se ^M 0.80
Br-82M	31	107	6.1 M	.976	0.024	0.0	
Br-82	31	108	35.4 H	1.0	0.0	0.0	
Kr-82	31	109	10 ¹⁵ y	1.0	0.0	0.0	Br ^M .976 → Br ^M 1.0 → Kr ^M 0.80

Isotope	Chain	ID	Half Life	BF(1)	BF(2)	BF(3)	Decay Chain
Ge-83	32	110	1.9 s	1.0	0.0	0.0	
As-83	32	111	13.0 s	.64	0.0	0.0	
Se-83M	32	112	70.0 s	0.0	1.0	0.0	
Se-83	32	113	22.5 M	1.0	0.0	0.0	
Br-83	32	114	2.39 H	1.0	0.0	0.0	
Kr-83M	32	115	1.86 H	1.0	0.0	0.0	
Kr-83	32	116	10 ¹⁵ y	1.0	0.0	0.0	
Sr-83	33	117	32.4 H	1.0	0.0	0.0	
Rb-83	33	118	86.2 D	.23	0.0	0.0	
Kr-83M	33	119	1.86 H	1.0	0.0	0.0	
Kr-83	33	120	10 ¹⁵ y	1.0	0.0	0.0	
Ge-84	34	121	1.2 s	1.0	0.0	0.0	
As-84	34	122	5.3 s	1.0	0.0	0.0	
Se-84	34	123	3.3 M	1.0	0.0	0.0	
Br-84M	34	124	6.0 M	0.0	1.0	0.0	
Br-84	34	125	31.8 M	1.0	0.0	0.0	
Kr-84	34	126	10 ¹⁵ y	1.0	0.0	0.0	
Rb-84M	35	127	20.5 M	0.5	0.5	0.0	
Rb-84	35	128	32.9 D	0.97	0.03	0.0	
Kr-84	35	129	10 ¹⁵ y	1.0	0.0	0.0	
Sr-84	35	130	10 ¹⁵ y	1.0	0.0	0.0	

Isotope	Chain	ID	Half Life	BF(1)	BF(2)	BF(3)	Decay Chain
As-85	36	131	2.03 s	0.0	1.0	0.0	
Se-85M	36	132	19.0 s	0.0	1.0	0.0	
Se-85	36	133	31.0 s	1.0	0.0	0.0	
Br-85	36	134	2.87 M	1.0	0.0	0.0	
Kr-85M	36	135	4.48 H	.2176	.7824	0.0	
Kr-85	36	136	10.73 Y	1.0	0.0	0.0	
Rb-85	36	137	10 ¹⁵ y	1.0	0.0	0.0	
Se-85M	37	138	19.0 s	.23	0.0	0.0	
Se-84	37	139	3.3 M	1.0	0.0	0.0	
Br-84M	37	140	6.0 M	1.0	0.0	0.0	
Kr-84	37	141	10 ¹⁵ y	1.0	0.0	0.0	
Y-85M	38	142	2.7 H	0.0	1.0	0.0	
Y-85	38	143	4.9 H	0.0	1.0	0.0	
Sr-85M	38	144	68.0 M	.86	.14	0.0	
Sr-85	38	145	64.8 D	1.0	0.0	0.0	
Rb-85	38	146	10 ¹⁵ y	1.0	0.0	0.0	
Se-86	39	147	16.6 s	0.0	1.0	0.0	
Br-86M	39	148	4.5 s	0.0	1.0	0.0	
Br-86	39	149	55 s	1.0	0.0	0.0	
Kr-86	39	150	10 ¹⁵ y	1.0	0.0	0.0	

Isotope	Chain	ID	Half Life	BF(1)	BF(2)	BF(3)	Decay Chain
Rb-86M	40	151	1.02 M	1.0	0.0	0.0	
Rb-86	40	152	18.8 D	1.0	0.0	0.0	Rb ^M 1.0 → Rb 1.0 → Sr
Sr-86	40	153	10 ¹⁵ Y	1.0	0.0	0.0	
Se-87	41	154	5.6 s	1.0	0.0	0.0	
Br-87	41	155	55.7 s	1.0	0.0	0.0	
Kr-87	41	156	76.0 M	0.0	1.0	0.0	
Sr-87M	41	157	2.81 H	.003	.997	0.0	
Rb-87	41	158	4.8x10 ¹⁰ Y	1.0	0.0	0.0	Se 1.0 → Br 1.0 → Kr 0.0 → Sr ^M .003 → Rb 1.0 → Sr
Sr-87	41	159	10 ¹⁵ Y	1.0	0.0	0.0	
Zr-87	42	160	1.6 H	1.0	0.0	0.0	
Y-87M	42	161	13.0 H	1.0	0.0	0.0	
Y-87	42	162	80.3 H	1.0	0.0	0.0	
Sr-87M	42	163	2.81 H	.003	.997	0.0	
Rb-87	42	164	4.8x10 ¹⁰ Y	1.0	0.0	0.0	Zr 1.0 → Y ^M 1.0 → Y 1.0 → Sr ^M .003 → Rb 1.0 → Sr
Sr-87	42	165	10 ¹⁵ Y	1.0	0.0	0.0	
Se-88	43	166	1.5 s	1.0	0.0	0.0	
Br-88	43	167	15.9 s	1.0	0.0	0.0	
Kr-88	43	168	2.8 H	1.0	0.0	0.0	
Rb-88	43	169	17.7 M	1.0	0.0	0.0	Se 1.0 → Br 1.0 → Kr 1.0 → Rb 1.0 → Sr
Sr-88	43	170	10 ¹⁵ Y	1.0	0.0	0.0	
Zr-88	44	171	85 D	1.0	0.0	0.0	
Y-88	44	172	106.6 D	1.0	0.0	0.0	Zr 1.0 → Y 1.0 → Sr
Sr-88	44	173	10 ¹⁵ Y	1.0	0.0	0.0	

Isotope	Chain	ID	Half Life	BF(1)	BF(2)	BF(3)	Decay Chain
Br-89	45	174	4.5 s	1.0	0.0	0.0	$\begin{array}{c} \text{Br} \xrightarrow{1.0} \text{Kr} \xrightarrow{1.0} \text{Rb} \xrightarrow{1.0} \text{Sr} \xrightarrow{.009} \text{Y} \xrightarrow{.99991} \text{Y} \\ \text{Zr} \xrightarrow{.94} \text{Zr} \xrightarrow{.998} \text{Y} \xrightarrow{1.0} \text{Y} \\ \text{Y} \xrightarrow{.06} \text{Y} \xrightarrow{.002} \text{Y} \end{array}$
Kr-89	45	175	3.16 M	1.0	0.0	0.0	
Rb-89	45	176	15.2 M	1.0	0.0	0.0	
Sr-89	45	177	50.5 D	.009	.99991	0.0	
Y-89M	45	178	15.7 s	1.0	0.0	0.0	
Y-89	45	179	10 ¹⁵ y	1.0	0.0	0.0	
Zr-89M	46	180	4.18 M	.94	0.0	0.6	$\begin{array}{c} \text{Zr} \xrightarrow{.94} \text{Zr} \xrightarrow{.998} \text{Y} \xrightarrow{1.0} \text{Y} \\ \text{Y} \xrightarrow{.06} \text{Y} \xrightarrow{.002} \text{Y} \end{array}$
Zr-89	46	181	78.5 H	.998	.002	0.0	
Y-89M	46	182	15.7 s	1.0	0.0	0.0	
Y-89	46	183	10 ¹⁵ y	1.0	0.0	0.0	
Br-90	47	184	1.6 s	1.0	0.0	0.0	$\begin{array}{c} \text{Br} \xrightarrow{1.0} \text{Kr} \xrightarrow{.11} \text{Rb} \xrightarrow{.05} \text{Sr} \xrightarrow{0.0} \text{Y} \xrightarrow{.996} \text{Y} \\ \text{Y} \xrightarrow{.89} \text{Y} \xrightarrow{.95} \text{Y} \xrightarrow{.004} \text{Y} \xrightarrow{0.0} \text{Y} \\ \text{Y} \xrightarrow{.89} \text{Y} \xrightarrow{.95} \text{Y} \xrightarrow{.004} \text{Y} \end{array}$
Kr-90	47	185	32.3 s	.11	.89	0.0	
Rb-90M	47	186	4.28 M	.05	.95	0.0	
Rb-90	47	187	2.7 M	1.0	0.0	0.0	
Sr-90	47	188	29 Y	0.0	1.0	0.0	
Y-90M	47	189	3.19 H	.996	.004	0.0	
Y-90	47	190	64.0 H	1.0	0.0	0.0	
Zr-90	47	191	10 ¹⁵ y	1.0	0.0	0.0	
Nb-90M	48	192	18.8 s	1.0	0.0	0.0	
Nb-90	48	193	14.59 H	1.0	0.0	0.0	
Zr-90	48	194	10 ¹⁵ y	1.0	0.0	0.0	

Isotope	Chain	ID	Half Life	BF(1)	BF(2)	BF(3)	Decay Chain
Mo-93M	54	217	6.9 H	1.0	0.0	0.0	
Mo-93	54	218	3.0×10^3 Y	.85	.15	0.0	Mo $\xrightarrow{1.0}$ Mo $\xrightarrow{.85}$ Nb $\xrightarrow{1.0}$ Nb
Nb-93M	54	219	13.6 Y	1.0	0.0	0.0	
Nb-93	54	220	10^{15} Y	1.0	0.0	0.0	Mo $\xrightarrow{1.0}$ Mo $\xrightarrow{.85}$ Nb $\xrightarrow{1.0}$ Nb
Rb-94	55	221	2.69 s	1.0	0.0	0.0	
Sr-94	55	222	1.29 M	1.0	0.0	0.0	
Y-94	55	223	19.0 M	1.0	0.0	0.0	Rb $\xrightarrow{1.0}$ Sr $\xrightarrow{1.0}$ Y $\xrightarrow{1.0}$ Zr
Zr-94	55	224	10^{15} Y	1.0	0.0	0.0	
Nb-94M	56	225	6.26 M	.998	.002	0.0	
Nb-94	56	226	2.0×10^4 Y	1.0	0.0	0.0	
Mo-94	56	227	10^{15} Y	1.0	0.0	0.0	Nb $\xrightarrow{.998}$ Nb $\xrightarrow{1.0}$ Mo
Sr-95	57	228	24.4 s	1.0	0.0	0.0	
Y-95	57	229	10.3 M	1.0	0.0	0.0	
Zr-95	57	230	64.0 D	.01	.99	0.0	
Nb-95M	57	231	87.0 H	1.0	0.0	0.0	
Nb-95	57	232	35.0 D	1.0	0.0	0.0	Sr $\xrightarrow{1.0}$ Y $\xrightarrow{1.0}$ Zr $\xrightarrow{.01}$ Nb $\xrightarrow{1.0}$ Nb $\xrightarrow{1.0}$ Mo
Mo-95	57	233	10^{15} Y	1.0	0.0	0.0	
Tc-95M	58	234	61.0 D	.04	.96	0.0	
Tc-95	58	235	20.0 H	1.0	0.0	0.0	
Mo-95	58	236	10^{15} Y	1.0	0.0	0.0	Tc $\xrightarrow{.04}$ Tc $\xrightarrow{1.0}$ Mo
Sr-96*	59	237	4.0 s	1.0	0.0	0.0	
Y-96	59	238	2.3 M	1.0	0.0	0.0	
Zr-96	59	239	10^{15} Y	1.0	0.0	0.0	Sr $\xrightarrow{1.0}$ Y $\xrightarrow{1.0}$ Zr

Isotope	Chain	ID	Half Life	BF(1)	BF(2)	BF(3)	Decay Chain
Nb-96	60	240	23.4 H	1.0	0.0	0.0	$\text{Nb} \xrightarrow{1.0} \text{Mo}$
Mo-96	60	241	10^{15} y	1.0	0.0	0.0	
Y-97	61	242	3.7 s	1.0	0.0	0.0	$\text{Y} \xrightarrow{1.0} \text{Zr} \xrightarrow{.94} \text{Nb} \xrightarrow{1.0} \text{Mo}$ $\text{Y} \xrightarrow{.06} \text{Nb} \xrightarrow{1.0} \text{Mo}$
Zr-97	61	243	16.9 H	.94	.06	0.0	
Nb-97M	61	244	1.0 M	1.0	0.0	0.0	
Nb-97	61	245	72.0 M	1.0	0.0	0.0	
Mo-97	61	246	10^{15} y	1.0	0.0	0.0	
Tc-97M	62	247	90.0 D	1.0	0.0	0.0	$\text{Tc} \xrightarrow{1.0} \text{Mo}$
Tc-97	62	248	2.6×10^6 y	1.0	0.0	0.0	
Mo-97	62	249	10^{15} y	1.0	0.0	0.0	
Y-98	63	250	2.0 s	1.0	0.0	0.0	$\text{Y} \xrightarrow{1.0} \text{Zr} \xrightarrow{1.0} \text{Nb} \xrightarrow{1.0} \text{Mo}$ $\text{Y} \xrightarrow{1.0} \text{Nb} \xrightarrow{1.0} \text{Mo}$
Zr-98	63	251	31.0 s	1.0	0.0	0.0	
Nb-98M	63	252	51.0 M	0.0	1.0	0.0	
Nb-98	63	253	2.9 s	1.0	0.0	0.0	
Mo-98	63	254	10^{15} y	1.0	0.0	0.0	
Tc-98	64	255	4.2×10^4 y	1.0	0.0	0.0	$\text{Tc} \xrightarrow{1.0} \text{Ru}$
Ru-98	64	256	10^{15} y	1.0	0.0	0.0	
Y-99	65	257	1.4 s	1.0	0.0	0.0	$\text{Y} \xrightarrow{1.0} \text{Zr} \xrightarrow{.30} \text{Nb} \xrightarrow{1.0} \text{Mo} \xrightarrow{.88} \text{Tc} \xrightarrow{1.0} \text{Ru}$ $\text{Y} \xrightarrow{.70} \text{Nb} \xrightarrow{1.0} \text{Mo} \xrightarrow{.88} \text{Tc} \xrightarrow{1.0} \text{Ru}$
Zr-99	65	258	2.1 s	.30	.70	0.0	
Nb-99M	65	259	2.6 M	1.0	0.0	0.0	
Nb-99	65	260	15.0 s	1.0	0.0	0.0	
Mo-99	65	261	66.02 H	.88	.12	0.0	
Tc-99M	65	262	6.02 H	1.0	0.0	0.0	
Tc-99	65	263	2.14×10^5 y	1.0	0.0	0.0	
Ru-99	65	264	10^{15} y	1.0	0.0	0.0	

Isotope	Chain	ID	Half Life	BF(1)	BF(2)	BF(3)	Decay Chain
Zr-100	66	265	7.1 s	1.0	0.0	0.0	
Nb-100	66	266	3.1 s	1.0	0.0	0.0	Zr $\xrightarrow{1.0}$ Nb $\xrightarrow{1.0}$ Mo
Mo-100	66	267	10^{15} Y	1.0	0.0	0.0	
Tc-100	67	268	16.0 s	1.0	0.0	0.0	
Ru-100	67	269	10^{15} Y	1.0	0.0	0.0	Tc $\xrightarrow{1.0}$ Ru
Zr-101	68	270	2.0 s	1.0	0.0	0.0	
Nb-101	68	271	7.0 s	1.0	0.0	0.0	
Mo-101	68	272	14.6 s	1.0	0.0	0.0	
Tc-101	68	273	14.3 M	1.0	0.0	0.0	Zr $\xrightarrow{1.0}$ Nb $\xrightarrow{1.0}$ Mo $\xrightarrow{1.0}$ Tc $\xrightarrow{1.0}$ Ru
Ru-101	68	274	10^{15} Y	1.0	0.0	0.0	
Rh-101M	69	275	4.34 D	.10	.90	0.0	
Rh-101	69	276	3.3 Y	1.0	0.0	0.0	
Ru-101	69	277	10^{15} Y	1.0	0.0	0.0	Rh $\xrightarrow{.10}$ Rh $\xrightarrow{1.0}$ Ru
Zr-102	70	278	2.9 s	1.0	0.0	0.0	
Nb-102	70	279	4.3 s	1.0	0.0	0.0	
Mo-102	70	280	11.0 M	1.0	0.0	0.0	
Tc-102M	70	281	4.4 M	0.0	1.0	0.0	
Tc-102	70	282	5.3 s	1.0	0.0	0.0	
Ru-102	70	283	10^{15} Y	1.0	0.0	0.0	Zr $\xrightarrow{1.0}$ Nb $\xrightarrow{1.0}$ Mo $\xrightarrow{1.0}$ Tc $\xrightarrow{0.0}$ Tc $\xrightarrow{1.0}$ Ru
Rh-102M	71	284	206 D	0.0	1.0	0.0	
Rh-102	71	285	2.9 Y	1.0	0.0	0.0	
Ru-102	71	286	10^{15} Y	1.0	0.0	0.0	Rh $\xrightarrow{0.0}$ Rh $\xrightarrow{1.0}$ Ru

Isotope	Chain	ID	Half Life	BF(1)	BF(2)	BF(3)	Decay Chain
Nb-103	72	287	1.5 s	1.0	0.0	0.0	
Mo-103	72	288	60.0 s	1.0	0.0	0.0	
Tc-103	72	289	50.0 s	1.0	0.0	0.0	
Ru-103	72	290	39.4 D	.99	.01	0.0	
Rh-103M	72	291	56.1 M	1.0	0.0	0.0	Nb-1.0 → Mo-1.0 → Tc-1.0 → Ru-.99 → Rh ^M 1.0 → Rh
Rh-103	72	292	10 ¹⁵ y	1.0	0.0	0.0	
Pd-103	73	293	17.5 D	.99	.01	0.0	
Rh-103M	73	294	56.1 M	1.0	0.0	0.0	
Rh-103	73	295	10 ¹⁵ y	1.0	0.0	0.0	Pd-.99 → Rh ^M 1.0 → Rh
Nb-104	74	296	4.8 s	1.0			
Mo-104	74	297	1.0 M	1.0			
Tc-104	74	298	18.1 M				Nb-1.0 → Mo-1.0 → Tc-1.0 → Ru
Ru-104	74	299	10 ¹⁵ y	1.0			
Rh-104M	75	300	4.34 M	.9982	.0018		
Rh-104	75	301	42.3 s	1.0			
Pd-104	75	302	10 ¹⁵ y	1.0			Rh ^M .9982 → Rh-1.0 → Pd
Nb-105	76	303	2.0 s	1.0			
Mo-105	76	304	36.0 s	1.0			
Tc-105	76	305	7.6 M	1.0			
Ru-105	76	306	4.44 H	.27	.73		
Rh-105M	76	307	45.0 s	1.0			
Rh-105	76	308	35.4 H	1.0			
Pd-105	76	309	10 ¹⁵ y	1.0			Nb-1.0 → Mo-1.0 → Tc-1.0 → Ru-.27 → Rh ^M 1.0 → Rh-1.0 → Pd

Isotope	Chain	ID	Half Life	BF(1)	BF(2)	BF(3)	Decay Chain
Mo-106	77	310	9.5 s	1.0			
Tc-106	77	311	36.0 s	1.0			
Ru-106	77	312	367 d	0.0	1.0		
Rh-106M	77	313	130 M				
Rh-106	77	314	29.8 s	1.0			
Pd-106	77	315	10 ¹⁵ y	1.0			$\begin{array}{c} \text{Mo} \xrightarrow{1.0} \text{Tc} \xrightarrow{1.0} \text{Ru} \xrightarrow{1.0} \text{Rh} \xrightarrow{0.0} \text{Pd} \\ \text{Rh} \xrightarrow{1.0} \text{Pd} \end{array}$
Ag-106M	78	316	8.5 D	0.0	1.0		
Ag-106	78	317	24.0 M	1.0	0.0		
Pd-106	78	318	10 ¹⁵ y	1.0	0.0		$\text{Ag}^M \longrightarrow \text{Ag} \longrightarrow \text{Pd}$
Mo-107	79	319	5.0 s	1.0	0.0		
Tc-107	79	320	21.2 s	1.0	0.0		
Ru-107	79	321	4.2 M	1.0	0.0		
Rh-107	79	322	21.7 M	0.0	1.0		
Pd-107M	79	323	21.3 s	1.0	0.0		
Pd-107	79	324	6.5x10 ⁶ y	0.0	1.0		
Ag-107M	79	325	44.3 s	1.0	0.0		
Ag-107	79	326	10 ¹⁵ y	0.0	0.0	0.0	$\begin{array}{c} \text{Mo} \xrightarrow{1.0} \text{Tc} \xrightarrow{1.0} \text{Ru} \xrightarrow{1.0} \text{Rh} \xrightarrow{0.0} \text{Pd} \xrightarrow{1.0} \text{Pd} \xrightarrow{0.0} \text{Ag} \xrightarrow{1.0} \text{Ag} \\ \text{Rh} \xrightarrow{1.0} \text{Pd} \end{array}$
Mo-108	80	327	1.1 s	1.0	0.0		
Tc-108	80	328	5.1 s	1.0	0.0		
Ru-108	80	329	4.5 M	0.0	1.0		
Rh-108M	80	330	16.8 s	0.0	1.0		
Rh-108	80	331	6.0 M	1.0	0.0		
Pd-108	80	332	10 ¹⁵ y	0.0	0.0	0.0	$\begin{array}{c} \text{Mo} \xrightarrow{1.0} \text{Tc} \xrightarrow{1.0} \text{Ru} \xrightarrow{0.0} \text{Rh} \xrightarrow{0.0} \text{Pd} \\ \text{Rh} \xrightarrow{1.0} \text{Pd} \end{array}$

Isotope	Chain	ID	Half Life	BF(1)	BF(2)	BF(3)	Decay Chain
Ag-108M		81	127 Y	.077	.923		
Ag-108		81	2.41 M	.023	.977		
Pd-108		81	10 ¹⁵ Y	0.0	0.0	0.0	Ag ^M .077 → Pd ^M 0.0 → Cd
Cd-108		81	10 ¹⁵ Y	0.0	0.0	0.0	Ag ^M .077 → Pd ^M 0.0 → Cd .977
TC-109		82	1.4 s	1.0	0.0	0.0	
Ru-109		82	34.0 s	.50	.50		
Rh-109M		82	50.0 s	1.0	0.0		
Rh-109		82	80.0 s	.50	.50		
Pd-109M		82	4.69 M	1.0	0.0	0.0	
Pd-109		82	13.43 H	.99995	.00005		
Ag-109M		82	39.8 s	1.0	0.0	0.0	
Ag-109		82	10 ¹⁵ Y	0.0	0.0	0.0	
Cd-109		83	453 D	1.0			
Ag-109M		83	39.8 s	1.0			
Ag-109		83	10 ¹⁵ Y	0.0	0.0	0.0	Cd ^M 1.0 → Ag ^M 1.0 → Ag
Ru-110		84	16.0 s	0.0	1.0		
Rh-110M		84	3.0 s	0.0	1.0		
Rh-110		84	28.0 s	1.0	0.0	0.0	
Pd-110		84	10 ¹⁵ Y	0.0	0.0	0.0	
							Ru ^M 0.0 → Rh ^M 0.0 → Pd 1.0
							Rh ^M 0.0 → Pd 1.0 → Rh 1.0
							Pd 1.0 → Rh 1.0 → Pd 1.0
							TC ^M 1.0 → Ru ^M .5 → Rh ^M 1.0 → Pd ^M 1.0 → Pd .99995
							Rh ^M .5 → Pd ^M 1.0 → Pd .99995
							Pd ^M 1.0 → Ag ^M 1.0 → Ag .00005

Isotope	Chain	ID	Half Life	BF(1)	BF(2)	BF(3)	Decay Chain
In-112M	88	368	20.9 M	1.0			
In-112	88	369	14.4 M	.56	.44		
Cd-112	88	370	10 ¹⁵ Y	0.0			
Sn-112	88	371	10 ¹⁵ Y	0.0			
Pd-113	89	372	1.5 M	0.10	0.90		
Ag-113M	89	373	1.15 M	0.0	0.045	0.955	
Ag-113	89	374	5.37 H	0.013	0.987		
Cd-113M	89	375	14 Y	0.001	0.0	0.999	
Cd-113	89	376	9x10 Y	0.0	1.0		
In-113M	89	377	99.5 M	1.0			
In-113	89	378	10 ¹⁵ Y	0.0			
Rh-114	90	379	1.7 s	1.0			
Pd-114	90	380	2.4 M	1.0			
Ag-114	90	381	4.5 s	1.0			
Cd-114	90	382	10 ¹⁵ Y	0.0			
In-114M	91	383	49.51 D	.965	0.0	.035	
In-114	91	384	71.9 s	.98	.02		
Sn-114	91	385	10 ¹⁵ Y	0.0			
Cd-114	91	386	10 ¹⁵ Y	0.0			

Isotope	Chain	ID	Half Life	BF(1)	BF(2)	BF(3)	Decay Chain
Pd-115	92	387	37 s	.27	.73		
Ag-115M	92	388	18 s	0.0	.73		
Ag-115	92	389	20 M	.085	.915		
Cd-115M	92	390	44.8 D	0.0	.99991		
Cd-115	92	391	53.4 H	1.0			
In-115M	92	392	4.49 H	.963	.037		
In-115	92	393	5.1x10 ¹⁵ y	1.0			
Sn-115	92	394	10 ¹⁵ y	0.0			
Pd-116	93	395	14 s	0.50	0.50		
Ag-116M	93	396	105 s	0.02	0.98		
Ag-116	93	397	2.68 M	1.0			
Cd-116	93	398	10 ¹⁵ y	0.0			
In-116M	94	399	54.1 M	0.0	1.0		
In-116	94	400	14.1 s	1.0			
Sn-116	94	401	10 ¹⁵ y	0.0			

Isotope	Chain	ID	Half Life	BF(1)	BF(2)	BF(3)	Decay Chain
Ag-117M	95	402	5.3 s	0.0	0.5	0.5	
Ag-117	95	403	1.21 M	0.20	0.8	0.0	
Cd-117M	95	404	3.4 H	0.0	0.44	0.56	
Cd-117	95	405	2.4 H	0.93	0.07	0.0	
In-117M	95	406	1.93 H	0.47	0.0	0.53	
In-117	95	407	42 M	0.0	1.0	0.0	
Sn-117M	95	408	14 D	1.0	0.0	0.0	
Sn-117	95	409	10 ¹⁵ y	0.0	0.0	0.0	
Pd-118	96	410	3.1 s	0.5	0.5	0.0	
Ag-118M	96	411	2.8 s	0.46	0.54	0.0	
Ag-118	96	412	3.7 s	1.0	0.0	0.0	
Cd-118	96	413	50.3 M	0.0	1.0	0.0	
In-118M	96	414	4.4 M	0.0	1.0	0.0	
In-118	96	415	5.0 s	1.0	0.0	0.0	
Sn-118	96	416	10 ¹⁵ y	0.0	0.0	0.0	
Sb-118M	97	417	5.0 H	0.0	1.0	0.0	
Sb-118	97	418	3.5 M	1.0	0.0	0.0	
Sn-118	97	419	10 ¹⁵ y	0.0	0.0	0.0	

Isotope	Chain	ID	Half Life	BF(1)	BF(2)	BF(3)	Decay Chain
Ag-119	98	420	2.1 s	0.5	0.5	0.0	
Cd-119M	98	421	1.9 M	0.0	0.5	0.0	
Cd-119	98	422	2.7 M	1.0	0.0	0.0	
In-119M	98	423	18.0 M	.05	0.0	.95	
In-119	98	424	2.1 M	.05	0.0	0.0	
Sn-119M	98	425	250 D	1.0	0.0	0.0	
Sn-119	98	426	10 ¹⁵ y	0.0	0.0	0.0	
Sb-119	99	427	38.1 H	1.0	0.0	0.0	Sb-119 → Sn
Sn-119	99	428	10 ¹⁵ y	0.0	0.0	0.0	
Ag-120	100	429	1.2 s	1.0	0.0	0.0	
Cd-120	100	430	50.8 s	0.50	0.50	0.0	
In-120M	100	431	3.0 s	0.0	1.0	0.0	
In-120	100	432	44.0 s	1.0	0.0	0.0	
Sn-120	100	433	10 ¹⁵ y	0.0	0.0	0.0	
Sb-120M	101	434	5.76 D	0.0	1.0	0.0	
Sb-120	101	435	16.0 M	1.0	0.0	0.0	
Sn-120	101	436	10 ¹⁵ y	0.0	0.0	0.0	
Ag-121	102	437	3.0 s	1.0	0.0	0.0	
Cd-121	102	438	12.8 s	0.18	.82	0.0	
In-121M	102	439	3.8 M	0.0	0.0	1.0	
In-121	102	440	30.0 s	0.0	1.0	0.0	
Sn-121M	102	441	55 Y	0.0	1.0	0.0	
Sn-121	102	442	27.1 H	1.0	0.0	0.0	
Sb-121	102	443	10 ¹⁵ y	0.0	0.0	0.0	

Isotope	Chain	ID	Half Life	BF(1)	BF(2)	BF(3)	Decay Chain
Te-121M	103	444	154 D	0.9	0.10	0.0	
Te-121	103	445	16.8 D	1.0	0.0	0.0	
Sb-121	103	446	10 ¹⁵ y	0.0	0.0	0.0	
Ag-122	104	447	1.5 s	1.0	0.0	0.0	
Cd-122	104	448	5.8 s	0.0	1.0	0.0	
In-122M	104	449	1.5 s	0.0	1.0	0.0	
In-122	104	450	9.2 s	1.0	0.0	0.0	
Sn-122	104	451	10 ¹⁵ y	0.0	0.0	0.0	
Sb-122M	105	452	4.2 M	1.0	0.0	0.0	
Sb-122	105	453	2.68 D	.03	.97	0.0	
Sn-122	105	454	10 ¹⁵ y	0.0	0.0	0.0	
Te-122	105	455	10 ¹⁵ y	0.0	0.0	0.0	
In-123M	106	456	48 s	0.0	0.5	0.5	
In-123	106	457	6.0 s	0.5	0.5	0.0	
Sn-123M	106	458	40.1 M	0.0	1.0	0.0	
Sn-123	106	459	129 D	1.0	0.0	0.0	
Sb-123	106	460	10 ¹⁵ y	0.0	0.0	0.0	
Te-123M	107	461	119.7 D	1.0	0.0	0.0	
Te-123	107	462	1.20x10 ¹⁵ y	1.0	0.0	0.0	
Sb-123	107	463	10 ¹⁵ y	0.0	0.0	0.0	

Isotope	Chain	ID	Half Life	BF(1)	BF(2)	BF(3)	Decay Chain
In-124	108	464	3.2 s	1.0	0.0	0.0	
Sn-124	108	465	10 ¹⁵ y	0.0	0.0	0.0	In $\xrightarrow{1.0}$ Sn
Sb-124M	109	466	20.2 M	1.0	0.0	0.0	
Sb-124	109	467	60.2 D	1.0	0.0	0.0	Sb ^M $\xrightarrow{1.0}$ Sb $\xrightarrow{1.0}$ Te
Te-124	109	468	10 ¹⁵ y	0.0	0.0	0.0	
In-125M	110	469	2.32 s	0.0	0.92	0.08	
In-125	110	470	12.2 s	0.70	0.30	0.0	
Sn-125M	110	471	9.5 M	0.0	1.0	0.0	
Sn-125	110	473	2.7 Y	.25	.75	0.0	
Te-125M	110	474	58 D	1.0	0.0	0.0	
Te-125	110	475	10 ¹⁵ y	0.0	0.0	0.0	
I-125	111	476	60.2 D	1.0	0.0	0.0	
Te-125	111	477	10 ¹⁵ y	0.0	0.0	0.0	I $\xrightarrow{1.0}$ Te
In-126	112	478	1.53 s	1.0	0.0	0.0	
Sn-126	112	479	1.0x10 ¹⁵ y	1.0	0.0	0.0	
Sb-126M	112	480	19.0 M	.14	.86	0.0	
Sb-126	112	481	12.4 D	1.0	0.0	0.0	
Te-126	112	482	10 ¹⁵ y	0.0	0.0	0.0	
I-126	113	483	13.0 D	.54	.46	0.0	
Te-126	113	484	10 ¹⁵ y	0.0	0.0	0.0	
Xe-126	113	485	10 ¹⁵ y	0.0	0.0	0.0	I $\xrightarrow{.54}$ Te $\xrightarrow{.46}$ Xe

Isotope	Chain	ID	Half Life	BF(1)	BF(2)	BF(3)	Decay Chain
In-127M	114	486	1.3 s	0.0	0.0	1.0	
In-127	114	487	3.7 s	0.0	1.0	0.0	
Sn-127M	114	488	4.1 M	0.0	1.0	0.0	
Sn-127	114	489	2.1 H	1.0	0.0	0.0	
Sb-127	114	490	3.9 D	.174	.826	0.0	
Te-127M	114	491	109D	.976	.024	0.0	
Te-127	114	492	9.4 H	1.0	0.0	0.0	
I-127	114	493	10 ¹⁵ y	0.0	0.0	0.0	
Xe-127M	115	494	69 s	1.0	0.0	0.0	
Xe-127	115	495	36.41 D	1.0	0.0	0.0	
I-127	115	496	10 ¹⁵ y	0.0	0.0	0.0	
In-128	116	497	3.7 s	1.0	0.0	0.0	
Sn-128	116	498	59.3 M	.998	.002	0.0	
Sb-128M	116	499	10.0 M	0.0	1.0	0.0	
Sb-128	116	500	9.1 H	1.0	0.0	0.0	
Te-128	116	501	10 ¹⁵ y	0.0	0.0	0.0	
I-128	117	502	24.99 M	.063	.937	0.0	
Te-128	117	503	1.5x10 ²⁴ y	1.0	0.0	0.0	
Xe-128	117	504	10 ¹⁵ y	0.0	0.0	0.0	

Isotope	Chain	ID	Half Life	BF(1)	BF(2)	BF(3)	Decay Chain
Sn-129M	118	505	7.5 M	0.0	1.0	0.0	
Sn-129	118	506	2.2 M	1.0	0.0	0.0	
Sb-129	118	507	4.4 H	.166	.834	0.0	
Te-129M	118	508	33.5 D	.636	.364	0.0	
Te-129	118	509	69 M	1.0	0.0	0.0	
I-129	118	510	1.6x10 ⁷ y	0.0	1.0	0.0	
Xe-129M	118	511	8.89 D	1.0	0.0	0.0	
Xe-129	118	512	10 ¹⁵ y	0.0	0.0	0.0	
Cs-129	119	513	32.3 H	1.0	0.0	0.0	
Xe-129	119	514	10 ¹⁵ y	0.0	0.0	0.0	
Sn-130	120	515	3.7 M	0.90	0.10	0.0	
Sb-130M	120	516	6.5 M	0.0	1.0	0.0	
Sb-130	120	517	40.0 M	1.0	0.0	0.0	
Te-130	120	518	2.0x10 ²¹ y	0.0	0.0	1.0	
I-130M	120	519	9.2 M	0.85	0.15	0.0	
I-130	120	520	10 ¹⁵ y	0.0	0.0	0.0	
Sn-131	121	522	63 s	1.0	0.0	0.0	
Sb-131	121	523	23.03 M	.07	.93	0.0	
Te-131M	121	524	30 H	.18	.82	0.0	
Te-131	121	525	25.0 M	1.0	0.0	0.0	
I-131	121	526	8.04 D	0.0	.014	.986	
Cs-131	121	527	9.688 D	1.0	0.0	0.0	
Xe-131M	121	528	11.77 D	1.0	0.0	0.0	
Xe-131	121	529	10 ¹⁵ y	0.0	0.0	0.0	

Isotope	Chain	ID	Half Life	BF(1)	BF(2)	BF(3)	Decay Chain
Sn-132	122	530	40 s	0.5	0.5	0.0	
Sb-132M	122	531	2.8 M	0.0	1.0	0.0	
Sb-132	122	532	4.2 M	1.0	0.0	0.0	
Te-132	122	533	78 H	0.0	1.0	0.0	
I-132M	122	534	83 M	.86	.14	0.0	
I-132	122	535	2.28 H	1.0	0.0	0.0	
Xe-132	122	536	10 ¹⁵ Y	0.0	0.0	0.0	
Cs-132	123	537	6.47 D	.978	.022	0.0	
Xe-132	123	538	10 ¹⁵ Y	0.0	0.0	0.0	
Ba-132	123	539	10 ¹⁵ Y	0.0	0.0	0.0	
Sb-133	124	540	2.7 M	.42	.58	0.0	
Te-133M	124	541	55.4 M	.13	0.0	.87	
Te-133	124	542	12.4 M	0.0	1.0	0.0	
I-133M	124	543	9.0 s	1.0	0.0	0.0	
I-133	124	544	20.9 H	.028	.972	0.0	
Xe-133M	124	545	2.23 D	1.0	0.0	0.0	
Xe-133	124	546	5.29 D	1.0	0.0	0.0	
Cs-133	124	547	10 ¹⁵ Y	0.0	0.0	0.0	
Ba-133M	125	548	38.9 H	1.0	0.0	0.0	
Ba-133	125	549	10.4 Y	1.0	0.0	0.0	
Cs-133	125	550	10 ¹⁵ Y	0.0	0.0	0.0	

1.0 Cs

Ba^M 1.0 → Ba 1.0 → Cs

Isotope Chain Te-134

Isotope	Chain	ID	Half Life	BF(1)	BF(2)	BF(3)	Decay Chain
Sb-134	126	551	10.4 s	1.0	0.0	0.0	
Te-134	126	552	42 M	0.0	1.0	0.0	
I-134M	126	553	3.5 M	.98	.02	0.0	
I-134	126	554	52.6 M	1.0	0.0	0.0	
Xe-134	126	555	10 ¹⁵ y	0.0	0.0	0.0	
Cs-134M	126	556	2.90 H	.99	.01	0.0	
Cs-134	126	557	2.062 Y	1.0	0.0	0.0	
Ba-134	126	558	10 ¹⁵ y	0.0	0.0	0.0	
Te-135	127	559	19.2 s	1.0	0.0	0.0	
I-135	127	560	6.61 M	.147	.853	0.0	
Xe-135M	127	561	15.6 M	1.0	0.0	0.0	
Xe-135	127	562	9.10 H	0.0	1.0	0.0	
Cs-135M	127	563	53 M	1.0	0.0	0.0	
Cs-135	127	564	3x10 ⁶ y	0.0	1.0	0.0	
Ba-135M	127	565	28.7 H	1.0	0.0	0.0	
Ba-135	127	566	10 ¹⁵ y	0.0	0.0	0.0	
La-135	128	567	19.4	1.0	0.0	0.0	
Ba-135	128	568	10 ¹⁵ y	0.0	0.0	0.0	
Te-136	129	569	17.5 s	0.0	1.0	0.0	
I-136M	129	570	46 s	0.0	1.0	0.0	
I-136	129	571	83 s	1.0	0.0	0.0	
Xe-136	129	572	10 ¹⁵ y	0.0	0.0	0.0	
Cs-136M	129	573	19 s	1.0	0.0	0.0	
Cs-136	129	574	13.1 D	1.0	0.0	0.0	
Ba-136	129	575	10 ¹⁵ y	0.0	0.0	0.0	

1.0

Sb-1.0 → Te-0.0 → I^M-.98 → I-1.0 → Xe-0.0 → Cs^M-.99 → Cs-1.0 → Ba

.01

.853

Te-1.0 → I-.147 → Xe^M1.0 → Xe-0.0 → Cs^M1.0 → Cs-0.0 → Ba^M1.0 → Ba

1.0

1.0

La-1.0 → Ba

1.0

Te-0.0 → I^M0.0 → I-1.0 → Xe-0.0 → Cs^M1.0 → Cs-1.0 → Ba

1.0

Isotope	Chain	ID	Half Life	BF(1)	BF(2)	BF(3)	Decay Chain
Te-137	130	576	4 s	1.0	0.0	0.0	
I-137	130	577	24.5 s	1.0	0.0	0.0	
Xe-137	130	578	3.82 M	1.0	0.0	0.0	
Cs-137	130	579	30.17 Y	.946	.054	0.0	
Ba-137M	130	580	2.551 M	1.0	0.0	0.0	Te-1.0 → I-1.0 → Xe-1.0 → Cs-.946 → Ba ^M → Ba
Ba-137	130	581	10 ¹⁵ Y	0.0	0.0	0.0	
Ce-137M	131	582	34.4 H	.994	.006	0.0	
Ce-137	131	583	9.0 H	1.0	0.0	0.0	
La-137	131	584	6.0x10 Y	1.0	0.0	0.0	
Ba-137	131	585	10 ¹⁵ Y	0.0	0.0	0.0	Ce ^M -.994 → Ce-1.0 → La-1.0 → Ba
Te-138	132	586	1.4 s	1.0	0.0	0.0	
I-138	132	587	6.5 s	1.0	0.0	0.0	
Xe-138	132	588	14.1 M	0.0	1.0	0.0	
Cs-138M	132	589	2.9 M	0.5	0.5	0.0	
Cs-138	132	590	32.2 M	1.0	0.0	0.0	Te-1.0 → I-1.0 → Xe-0.0 → Cs ^M -.50 → Ba
Ba-138	132	591	10 ¹⁵ Y	0.0	0.0	0.0	
La-138	133	592	1.1x10 ¹¹ Y	.30	.70	0.0	
Ce-138	133	593	10 ¹⁵ Y	0.0	0.0	0.0	
Ba-138	133	594	10 ¹⁵ Y	0.0	0.0	0.0	La-.30 → Ce-0.0 → Ba
I-139	134	595	2.3 s	1.0	0.0	0.0	
Xe-139	134	596	39.7 s	1.0	0.0	0.0	
Cs-139	134	597	9.5 M	1.0	0.0	0.0	
Ba-139	134	598	82.9 M	1.0	0.0	0.0	
La-139	134	599	10 ¹⁵ Y	0.0	0.0	0.0	I-1.0 → Xe-1.0 → Cs-1.0 → Ba-1.0 → La

Isotope	Chain	ID	Half Life	BF(1)	BF(2)	BF(3)	Decay Chain
Pr-139	135	600	4.4 H	0.0	1.0	0.0	
Ce-139M	135	601	56 s	1.0	0.0	0.0	
Ce-139	135	602	137.2 D	1.0	0.0	0.0	
La-139	135	603	10 ¹⁵ y	0.0	0.0	0.0	
Xe-140	136	604	14 s	1.0	0.0	0.0	
Cs-140	136	605	65 s	1.0	0.0	0.0	
Ba-140	136	606	12.79 D	1.0	0.0	0.0	
La-140	136	607	40.3 H	0.0	1.0	0.0	
Pr-140	136	608	3.39 M	1.0	0.0	0.0	
Ce-140	136	609	10 ¹⁵ y	0.0	0.0	0.0	
Xe-141	137	610	1.83 s	1.0	0.0	0.0	
Cs-141	137	611	24.9 s	1.0	0.0	0.0	
Ba-141	137	612	18.2 M	1.0	0.0	0.0	
La-141	137	613	3.9 H	1.0	0.0	0.0	
Ce-141	137	614	32.5 D	0.0	1.0	0.0	
Nd-141M	137	615	61 s	1.0	0.0	0.0	
Nd-141	137	616	2.5 H	1.0	0.0	0.0	
Pr-141	137	617	10 ¹⁵ y	0.0	0.0	0.0	
Xe-142	138	618	1.2 s	1.0	0.0	0.0	
Cs-142	138	619	1.69 s	1.0	0.0	0.0	
Ba-142	138	620	10.6 M	1.0	0.0	0.0	
La-142	138	621	93 M	1.0	0.0	0.0	
Ce-142	138	622	10 ¹⁵ y	0.0	0.0	0.0	

Isotope	Chain	ID	Half Life	BF(1)	BF(2)	BF(3)	Decay Chain
Pr-142M	139	623	14.6 M	1.0	0.0	0.0	
Pr-142	139	624	19.2 H	1.0	0.0	0.0	
Nd-142	139	625	10 ¹⁵ Y	0.0	0.0	0.0	Pr ^M 1.0 → Pr 1.0 → Nd
Cs-143	140	626	1.78 s	1.0	0.0	0.0	
Ba-143	140	627	13.5 s	1.0	0.0	0.0	
La-143	140	628	14.0 M	1.0	0.0	0.0	
Ce-143	140	629	33.0 H	1.0	0.0	0.0	
Pr-143	140	630	13.58 D	1.0	0.0	0.0	
Nd-143	140	631	10 ¹⁵ Y	0.0	0.0	0.0	Cs 1.0 → Ba 1.0 → La 1.0 → Ce 1.0 → Pr 1.0 → Nd
Xe-144	141	632	1.2 s	1.0	0.0	0.0	
Cs-144	141	633	1.0 s	1.0	0.0	0.0	
Ba-144	141	634	11.9 s	1.0	0.0	0.0	
La-144	141	635	40.0 s	1.0	0.0	0.0	
Ce-144	141	636	284 d	.005	.995	0.0	
Pr-144M	141	637	7.2 M	.9995	.0005	0.0	
Pr-144	141	638	17.3 M	1.0	0.0	0.0	
Nd-144	141	639	10 ¹⁵ Y	0.0	0.0	0.0	Xe 1.0 → Cs 1.0 → Ba 1.0 → La 1.0 → Ce .005 → Pr ^M .9995 → Pr .995 1.0 → Nd .0005
Pm-144	142	640	349 D	1.0	0.0	0.0	
Nd-144	142	641	10 ¹⁵ Y	0.0	0.0	0.0	Pm 1.0 → Nd
Ba-145	143	642	5 s	1.0	0.0	0.0	
La-145	143	643	30 s	1.0	0.0	0.0	
Ce-145	143	644	3.0 M	1.0	0.0	0.0	
Pr-145	143	645	5.98 H	1.0	0.0	0.0	
Nd-145	143	646	10 ¹⁵ Y	0.0	0.0	0.0	Ba 1.0 → La 1.0 → Ce 1.0 → Pr 1.0 → Nd

Isotope	Chain	ID	Half Life	BF(1)	BF(2)	BF(3)	Decay Chain
Pm-145	144	647	17.7 Y	1.0	0.0	0.0	
Nd-145	144	648	10^{15} Y	0.0	0.0	0.0	Pm-1.0 → Nd
Ba-146	145	649	1.7 s	1.0	0.0	0.0	
La-146	145	650	11 s	1.0	0.0	0.0	
Ce-146	145	651	14 M	1.0	0.0	0.0	
Pr-146	145	652	24.0 M	1.0	0.0	0.0	Ba-1.0 → La-1.0 → Ce-1.0 → Pr-1.0 → Nd
Nd-146	145	653	10^{15} Y	0.0	0.0	0.0	
Pm-146	146	654	5.5 Y	.35	.65	0.0	
Sm-146	146	655	1.03×10^1 Y	0.0	0.0	0.0	
Nd-146	146	656	10^{15} Y	0.0	0.0	0.0	Pm-0.35 → Sm-0.0 → Nd
Ce-147	147	657	56 s	1.0	0.0	0.0	
Pr-147	147	658	13 M	1.0	0.0	0.0	
Nd-147	147	659	11.0 D	1.0	0.0	0.0	
Pm-147	147	660	2.6234 Y	1.0	0.0	0.0	Ce-1.0 → Pr-1.0 → Nd-1.0 → Pm-1.0 → Sm
Sm-147	147	661	1.06×10^{11} Y	1.0	0.0	0.0	
La-148	148	662	1.3 s	1.0	0.0	0.0	
Ce-148	148	663	48 s	1.0	0.0	0.0	
Pr-148	148	664	2.3 M	1.0	0.0	0.0	La-1.0 → Ce-1.0 → Pr-1.0 → Nd
Nd-148	148	665	10^{15} Y	0.0	0.0	0.0	
Pm-148M	149	666	41.3 D	.046	.954	0.0	
Pm-148	149	667	5.37 D	1.0	0.0	0.0	
Sm-148	149	668	8×10^{15} Y	1.0	0.0	0.0	Pm-0.046 → Pm-1.0 → Sm

Isotope	Chain	ID	Half Life	BF(1)	BF(2)	BF(3)	Decay Chain
Ce-149	150	669	5.0 s	1.0	0.0	0.0	
Pr-149	150	670	2.3 M	1.0	0.0	0.0	
Nd-149	150	671	1.73 M	1.0	0.0	0.0	
Pm-149	150	672	53.1 H	1.0	0.0	0.0	Ce-1.0 → Pr-1.0 → Nd-1.0 → Pm-1.0 → Sm
Sm-149	150	673	10 ¹⁵ y	0.0	0.0	0.0	
Eu-149	151	674	93.1 D	1.0	0.0	0.0	
Sm-149	151	675	10 ¹⁵ y	0.0	0.0	0.0	Eu-1.0 → Sm
Ce-150	152	676	4 s	1.0	0.0	0.0	
Pr-150	152	677	6.2 s	1.0	0.0	0.0	
Nd-150	152	578	10 ¹⁵ y	0.0	0.0	0.0	Ce-1.0 → Pr-1.0 → Nd
Pm-150	153	679	2.68 H	1.0	0.0	0.0	
Sm-150	153	680	10 ¹⁵ y	0.0	0.0	0.0	Pm-1.0 → Sm
Ce-151	154	681	1.0 s	1.0	0.0	0.0	
Pr-151	154	682	4 s	1.0	0.0	0.0	
Nd-151	154	683	12.4 M	1.0	0.0	0.0	
Pm-151	154	684	28.4 H	1.0	0.0	0.0	
Sm-151	154	685	90 Y	1.0	0.0	0.0	
Eu-151	154	686	10 ¹⁵ y	10 ¹⁵ y	0.0	0.0	Ce-1.0 → Pr-1.0 → Nd-1.0 → Pm-1.0 → Sm-1.0 → Eu
Nd-152	155	687	11.4 M	0.0	1.0	0.0	
Pm-152M	155	688	18 M	0.0	1.0	0.0	
Pm-152	155	689	4.1 M	1.0	0.0	0.0	
Sm-152	155	690	10 ¹⁵ y	0.0	0.0	0.0	Nd-0.0 → Pm ^M -0.0 → Sm-1.0 → Nd-1.0 → Pm-1.0 → Sm-1.0

Isotope	Chain	ID	Half Life	BF(1)	BF(2)	BF(3)	Decay Chain
Eu-152M	156	691	9.3 H	0.0	.23	.77	
Eu-152	156	692	13 Y	.72	.28	0.0	
Sm-152	156	693	10 ¹⁵ Y	0.0	0.0	0.0	
Gd-152	156	694	1.1x10 ¹⁴ Y	1.0	0.0	0.0	
Pm-153	157	695	5.4 M	1.0	0.0	0.0	
Sm-153	157	696	46.8 H	1.0	0.0	0.0	
Eu-153	157	697	10 ¹⁵ Y	0.0	0.0	0.0	
Gd-153	158	698	241.6 D	1.0	0.0	0.0	
Eu-153	158	699	10 ¹⁵ Y	0.0	0.0	0.0	
Nd-154	159	700	40 s	0.0	1.0	0.0	
Pm-154M	159	701	1.7 M	1.0	0.0	0.0	
Pm-154	159	702	2.7 M	1.0	0.0	0.0	
Sm-154	159	703	10 ¹⁵ Y	0.0	0.0	0.0	
Eu-154M	160	704	46 M	1.0	0.0	0.0	
Eu-154	160	705	8.5 Y	.0002	.9998	0.0	
Sm-154	160	706	10 ¹⁵ Y	0.0	0.0	0.0	
Gd-154	160	707	10 ¹⁵ Y	0.0	0.0	0.0	

Isotope	Chain	ID	Half Life	BF(1)	BF(2)	BF(3)	Decay Chain
Sm-155	161	708	22.4 M	1.0	0.0	0.0	
Eu-155	161	709	4.9 Y	1.0	0.0	0.0	Sm-1.0 → Eu-1.0 → Gd
Gd-155	161	710	10 ¹⁵ Y	0.0	0.0	0.0	
Tb-155	162	711	5.3 D	1.0	0.0	0.0	
Gd-155	162	712	10 ¹⁵ Y	0.0	0.0	0.0	Tb-1.0 → Gd
Sm-156	163	713	9.4 H	1.0	0.0	0.0	
Eu-156	163	714	15 d	1.0	0.0	0.0	
Gd-156	163	715	10 ¹⁵ Y	0.0	0.0	0.0	Sm-1.0 → Eu-1.0 → Gd
Tb-156M	164	716	5.0 H	.50	.50	0.0	
Tb-156	164	717	5.3 D	1.0	0.0	0.0	
Gd-156	164	718	10 ¹⁵ Y	0.0	0.0	0.0	Tb ^M -1.0 → Gd
							← .50
Sm-157	165	719	8.0 M	1.0	0.0	0.0	
Eu-157	165	720	15.13 H	1.0	0.0	0.0	
Gd-157	165	721	10 ¹⁵ Y	0.0	0.0	0.0	Sm-1.0 → Eu-1.0 → Gd
Tb-157	166	722	150 Y	1.0	0.0	0.0	
Gd-157	166	723	10 ¹⁵ Y	0.0	0.0	0.0	Tb-1.0 → Gd
Eu-158	167	724	45.9 M	1.0	0.0	0.0	
Gd-158	167	725	10 ¹⁵ Y	0.0	0.0	0.0	Eu-1.0 → Gd

Isotope	Chain	ID	Half Life	BF(1)	BF(2)	BF(3)	Decay Chain
Tb-158M	168	726	10.5 s	1.0	0.0	0.0	
Tb-158	168	727	150 Y	.84	.16	0.0	
Gd-158	168	728	10 ¹⁵ Y	0.0	0.0	0.0	
DY-158	168	729	10 ¹⁵ Y	0.0	0.0	0.0	Tb ^M $\xrightarrow{1.0}$ Tb $\xrightarrow{.84}$ Gd $\xrightarrow{0.0}$ DY
Eu-159	169	730	18.1 M	1.0	0.0	0.0	
Gd-159	169	731	18.6 H	1.0	0.0	0.0	
Tb-159	169	732	10 ¹⁵ Y	0.0	0.0	0.0	Eu $\xrightarrow{1.0}$ Gd $\xrightarrow{1.0}$ Tb
DY-159	170	733	144.4 D	1.0	0.0	0.0	
Tb-159	170	734	10 ¹⁵ Y	0.0	0.0	0.0	DY $\xrightarrow{1.0}$ Tb
Eu-160	171	735	48 s	1.0	0.0	0.0	
Gd-160	171	736	10 ¹⁵ Y	0.0	0.0	0.0	Eu $\xrightarrow{1.0}$ Gd
Tb-160	172	737	72.1 D	1.0	0.0	0.0	
DY-160	172	738	10 ¹⁵ Y	0.0	0.0	0.0	Tb $\xrightarrow{1.0}$ DY
Gd-161	173	739	3.7 M	1.0	0.0	0.0	
Tb-161	173	740	6.9 D	1.0	0.0	0.0	
DY-161	173	741	10 ¹⁵ Y	0.0	0.0	0.0	Gd $\xrightarrow{1.0}$ Tb $\xrightarrow{1.0}$ DY
Ho-161M	174	742	6.7 s	1.0	0.0	0.0	
Ho-161	174	743	2.48 H	1.0	0.0	0.0	
DY-161	174	744	10 ¹⁵ Y	0.0	0.0	0.0	Ho ^M $\xrightarrow{1.0}$ Ho $\xrightarrow{1.0}$ DY

Isotope	Chain	ID	Half Life	BF(1)	BF(2)	BF(3)	Decay Chain
Gd-162	175	745	9.0 M	.02	.98	0.0	
Tb-162M	175	746	2.23 H	0.0	1.0	0.0	
Tb-162	175	747	7.47 M	1.0	0.0	0.0	
Dy-162	175	748	10 ¹⁵ y	0.0	0.0	0.0	
Ho-162	176	749	68 M	.63	.37	0.0	
HO-162	176	750	15 M	1.0	0.0	0.0	
DY-162	176	751	10 ¹⁵ y	0.0	0.0	0.0	
Tb-163	177	752	19.5 M	1.0	0.0	0.0	
DY-163	177	753	10 ¹⁵ y	0.0	0.0	0.0	
HO-163M	178	754	1.09 s	1.0	0.0	0.0	
HO-163	178	755	33 Y	1.0	0.0	0.0	
DY-163	178	756	10 ¹⁵ y	0.0	0.0	0.0	
Tb-164	179	757	3.0 M	1.0	0.0	0.0	
DY-164	179	758	10 ¹⁵ y	0.0	0.0	0.0	
HO-164M	180	759	37 M	1.0	0.0	0.0	
HO-164	180	760	29.0 M	.53	.47	0.0	
Er-164	180	761	10 ¹⁵ y	0.0	0.0	0.0	
DY-164	180	762	10 ¹⁵ y	0.0	0.0	0.0	

Isotope	Chain	ID	Half Life	BF(1)	BF(2)	BF(3)	Decay Chain
Dy-165M	181	763	1.26 M	.975	.025	0.0	
DY-165	181	764	2.33 H	1.0	0.0	0.0	
HO-165	181	765	10 ¹⁵ Y	0.0	0.0	0.0	
Er-165	182	766	10.4 H	1.0	0.0	0.0	
HO-165	182	767	10 ¹⁵ Y	0.0	0.0	0.0	
DY-166	183	768	81.5 H	0.0	1.0	0.0	
HO-166M	183	769	1.2x10 ³ Y	0.0	1.0	0.0	
HO-166	183	770	26.8 H	1.0	0.0	0.0	
Er-166	183	771	10 ¹⁵ Y	0.0	0.0	0.0	
Tm-166	184	772	7.7 H	1.0	0.0	0.0	
Er-166	184	773	10 ¹⁵ Y	0.0	0.0	0.0	
DY-167	185	774	6.2 M	1.0	0.0	0.0	
HO-167	185	775	3.1 H	0.10	0.90	0.0	
ER-167M	185	776	2.28 S	1.0	0.0	0.0	
ER-167	185	777	10 ¹⁵ Y	0.0	0.0	0.0	
Tm-167	186	778	9.25 D	.98	.02	0.0	
ER-167M	186	779	2.28 S	1.0	0.0	0.0	
ER-167	186	780	10 ¹⁵ Y	0.0	0.0	0.0	

Isotope	Chain	ID	Half Life	BF(1)	BF(2)	BF(3)	Decay Chain
Ho-168	187	781	3.0 M	0.0	1.0	0.0	
Tm-168	187	782	43.1 D	1.0	0.0	0.0	
Er-168	187	783	10 ¹⁵ Y	0.0	0.0	0.0	
Ho-169	188	784	4.6 M	1.0	0.0	0.0	
Er-169	188	785	9.4 D	1.0	0.0	0.0	
Tm-169	188	786	10 ¹⁵ Y	0.0	0.0	0.0	
Ho-170M	189	787	43 S	0.0	1.0	0.0	
Ho-170	189	788	2.8 M	1.0	0.0	0.0	
Er-170	180	789	10 ¹⁵ Y	0.0	0.0	0.0	
Tm-170	190	790	128.6 D	.998	.002	0.0	
Tb-170	190	791	10 ¹⁵ Y	0.0	0.0	0.0	
Er-170	190	792	10 ¹⁵ Y	0.0	0.0	0.0	
Er-171	191	793	7.52 H	1.0	0.0	0.0	
Tm-171	191	794	1.92 Y	1.0	0.0	0.0	
Yb-171	191	795	10 ¹⁵ Y	0.0	0.0	0.0	
Er-172	192	796	49.5 H	1.0	0.0	0.0	
Tm-172	192	797	63.6 H	1.0	0.0	0.0	
Yb-172	192	798	10 ¹⁵ Y	0.0	0.0	0.0	

APPENDIX B

Fission yield values are given in the following pages. The first column represents the isotope I.D. numbers and the ten numbers in each row are the fission yield values for the following:

U-235T, U-235F, U-235HE, U-238F, U-238HE, Pu-239T, Pu-239F,
Pu-241T, U-233T, Th-232F.

VITA

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